# What equations describe the Index ellipsoid?

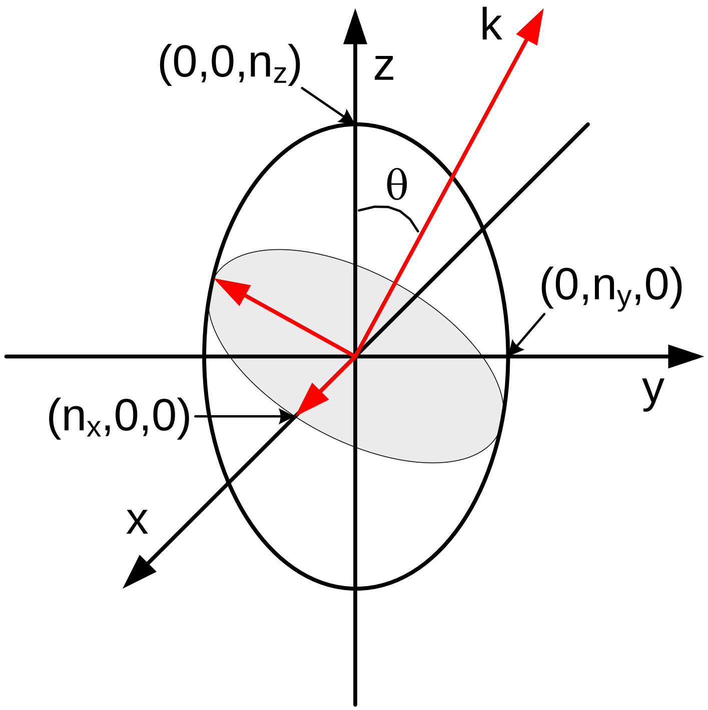
$$\frac{x'^2}{n_{x'}^2} + \frac{y'^2}{n_{y'}^2} + \frac{z'^2}{n_{z'}^2} = 1$$

If x', y', and z' are principal axes of the crystal.

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2$$
$$+\left(\frac{1}{n^2}\right)_4 2yz + \left(\frac{1}{n^2}\right)_5 2xz + \left(\frac{1}{n^2}\right)_6 2xy = 1$$

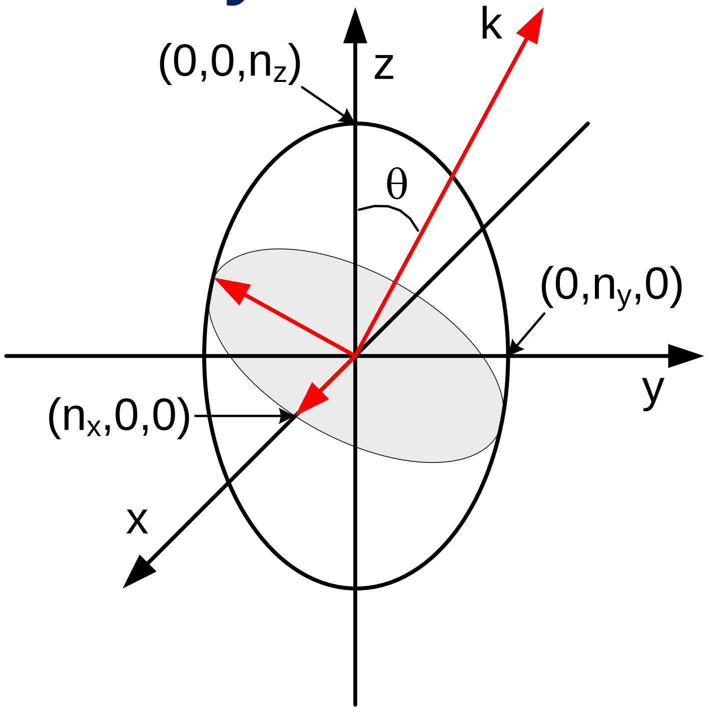
For arbitrary axes x, y, and z

# How do we use the index ellipsoid?



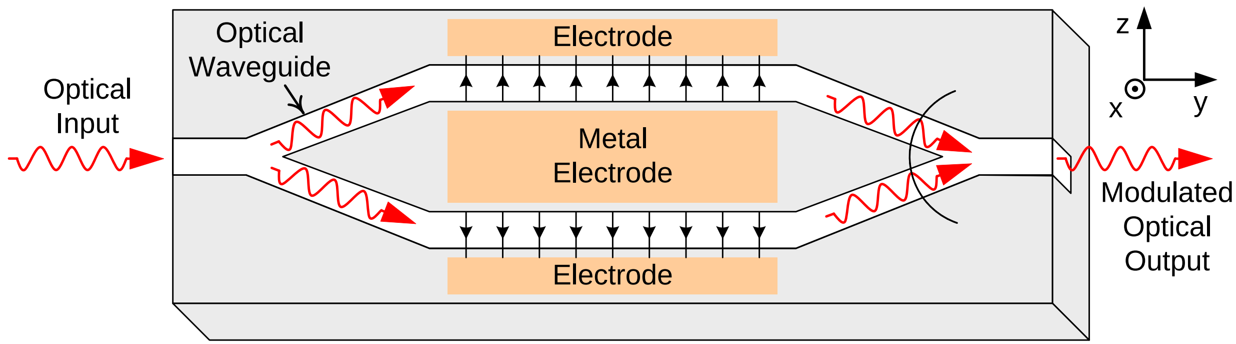

- The waves that can propagate as linearly polarized waves have polarization along the major and minor axes of the ellipse perpendicular to the wavevector **k**.

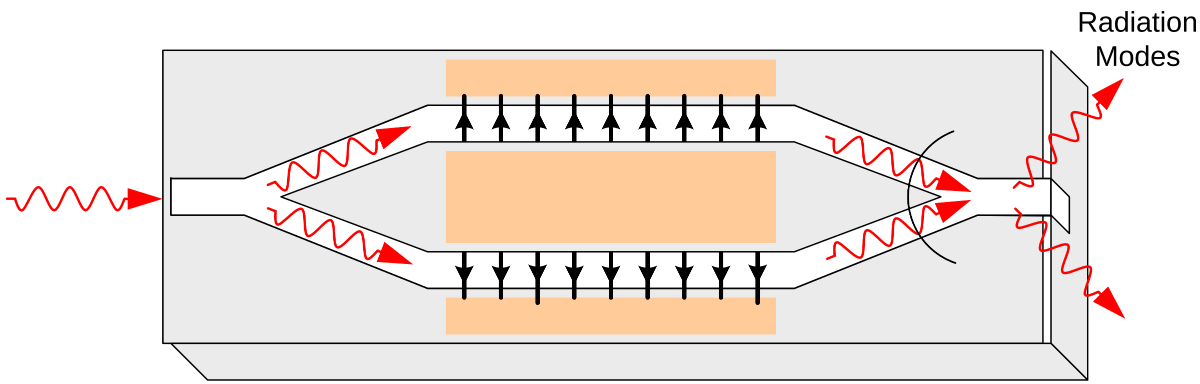
# What is special about uni-axial crystals?




- $n_x = n_y = n_o$
- There is always a linearly polarized wave, called the ordinary wave, that "sees" a refractive index $n_o$, regardless of the direction of propagation.

# What changes in the modulator to cause constructive interference to become destructive?




- The magnitude and/or direction of the applied electric fields.

# Are there any important corrections to the notes?

$$n_0(E_z) = n_0 - \frac{1}{2} n_o^2 r_{13} E_z$$
$$n_e(E_z) = n_e - \frac{1}{2} n_o^2 r_{33} E_z$$

$$n_o(E_z) = n_o - \frac{1}{2} n_o^3 r_{13} E_z$$
$$n_e(E_z) = n_e - \frac{1}{2} n_e^3 r_{33} E_z$$

- A large refractive index enhances the electro-optic effect.

# How did we get the equations for $LiNbO_3$?

$$\begin{bmatrix} \Delta\left(\frac{1}{n^2}\right)_1 \\ \Delta\left(\frac{1}{n^2}\right)_2 \\ \Delta\left(\frac{1}{n^2}\right)_3 \\ \Delta\left(\frac{1}{n^2}\right)_4 \\ \Delta\left(\frac{1}{n^2}\right)_5 \\ \Delta\left(\frac{1}{n^2}\right)_6 \end{bmatrix} = \begin{bmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ E_z \end{bmatrix} \Rightarrow \begin{matrix} \Delta\left(\frac{1}{n^2}\right)_1 = \Delta\left(\frac{1}{n^2}\right)_2 = r_{13}E_z \\ \Delta\left(\frac{1}{n^2}\right)_3 = r_{33}E_z \end{matrix}$$

# How did we get the equations for $LiNbO_3$?

$$\Delta\left(\frac{1}{n^2}\right)_3 = r_{33}E_z$$

taking a derivative we find:

$$\Delta\left(\frac{1}{n^2}\right)_3 = \Delta\frac{1}{n_e{}^2} = -2\frac{1}{n_e{}^3}\Delta n_e$$

$$\Rightarrow \Delta n_e = -\frac{1}{2}n_e{}^3 r_{33}E_z \text{ and } n_e\left(E_z\right) = n_e - \frac{1}{2}n_e^3 r_{33}E_z$$

# How does $LiNbO_3$ compare with other electro-optic materials?

**Table 17.1** Linear electrooptic coefficients for some relevant crys

| Material | Symmetry | Wavelength ($\mu$m) | Electrooptic c ($10^{-12}$ m/ |
|---|---|---|---|
| $LiNbO_3$ | $3m$ | 0.632 | $r_{13}$ = 9. |
| | | | $r_{22}$ = 6. |
| | | | $r_{33}$ = 3 |
| | | | $r_{51}$ = 32 |
| $LiIO_3$ | 6 | 0.633 | $r_{13}$ = 4. |
| | | | $r_{41}$ = 1. |
| GaAs | $\bar{4}3m$ | 0.9 | $r_{41}$ = 1. |
| | | 1.15 | $r_{41}$ = 1. |
| KDP | $\bar{4}2m$ | 0.633 | $r_{63}$ = 11 |
| | | | $r_{41}$ = 8 |
| ADP | $\bar{4}2m$ | 0.633 | $r_{63}$ = 8. |
| | | | $r_{41}$ = 2 |
| Quartz | 32 | $\approx$ 0.632 | $r_{41}$ = 0. |
| | | | $r_{63}$ = 0. |
| $BaTiO_3$ | $4mm$ | $\approx$ 0.632 | $r_{33}$ = 2 |
| | | | $r_{13}$ = 8 |
| | | | $r_{42}$ = 8 |
| $LiTaO_3$ | $3m$ | $\approx$ 0.632 | $r_{33}$ = 3 |
| | | | $r_{13}$ = 5. |

From "Optoelectronics" by Pollock

# Can we have a quadratic electro-optic effect?

$$\underbrace{n(E)}_{\text{Refractive Index}} = n_0 + \underbrace{a_1 E}_{\substack{\textit{Linear Electro-optic Effect} \\ \textit{(Pockels Effect)}}} + \underbrace{a_2 E^2}_{\textit{Kerr Effect}}$$

**Friedrich Pockels (1865–1913)** was first to describe the linear electro-optic effect in 1893.

**John Kerr (1824–1907)** discovered the quadratic electro-optic effect in 1875.