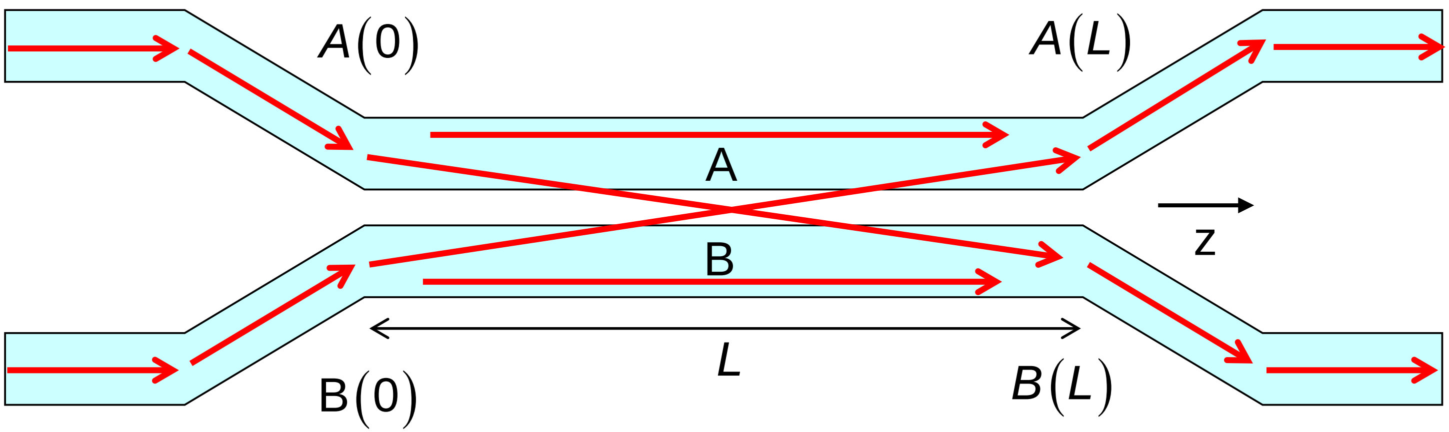
Coupled Optical Waveguides

$$E_A = A(z) E_A(x) e^{-i\beta z},$$
$$E_B = B(z) E_B(x) e^{-i\beta z}$$

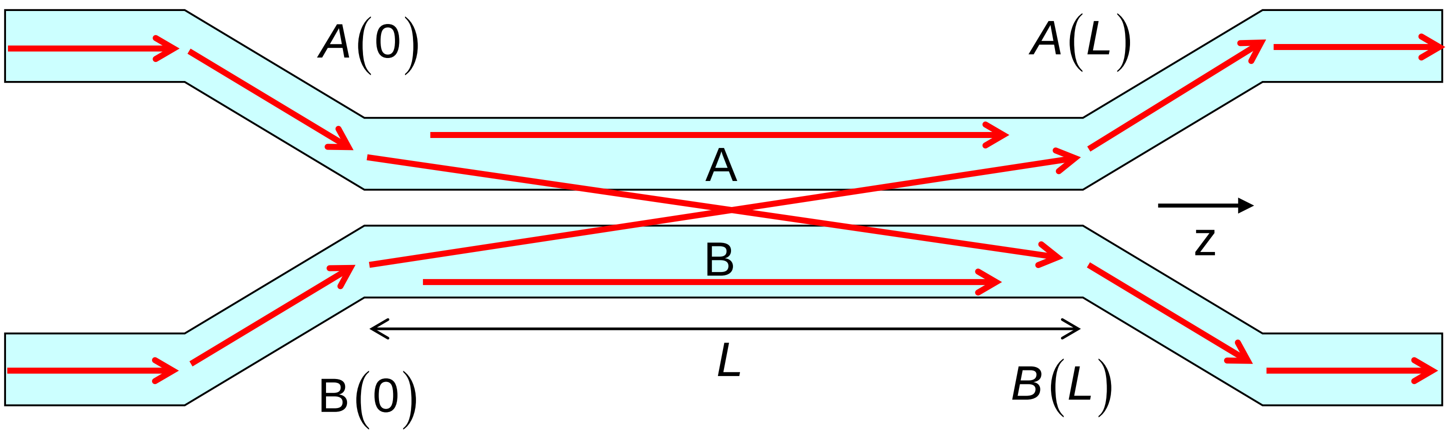
$$A(z) = \cos\left(\frac{\beta_e - \beta_0}{2} z\right) A(0) - j \sin\left(\frac{\beta_e - \beta_0}{2} z\right) B(0)$$
$$\mathrm{B}(z) = -j \sin\left(\frac{\beta_e - \beta_0}{2} z\right) A(0) + \cos\left(\frac{\beta_e - \beta_0}{2} z\right) B(0)$$



Coupled Optical Waveguides

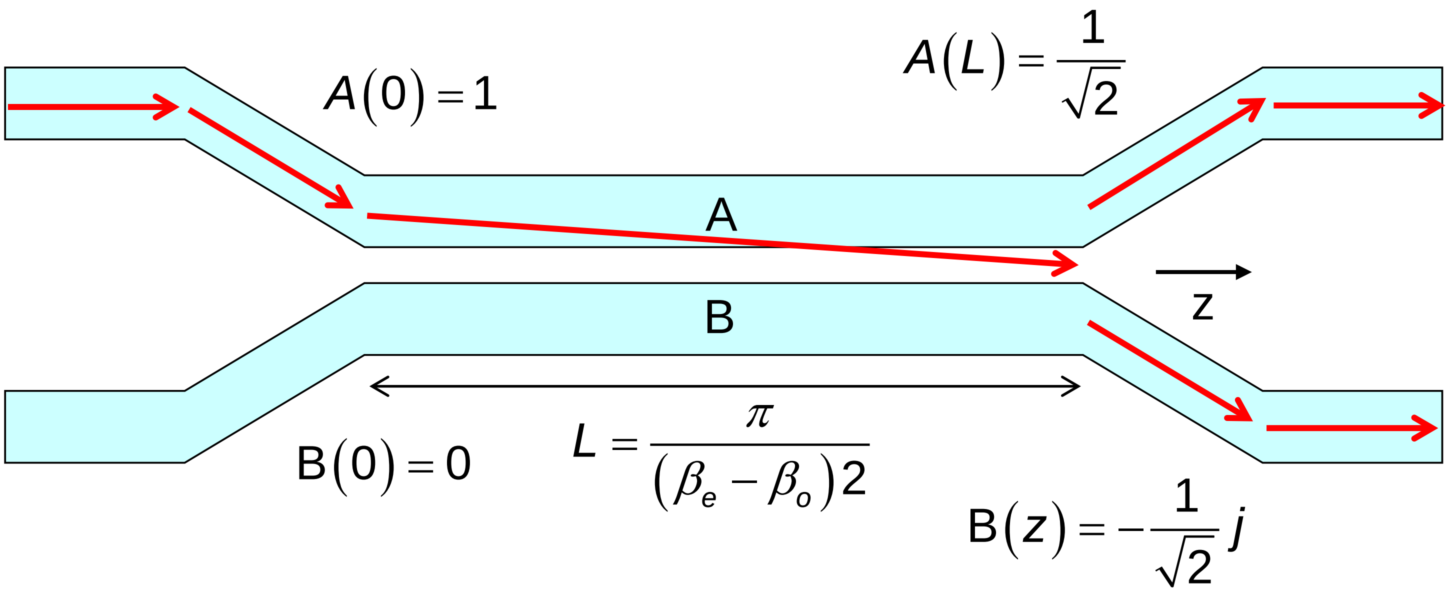
$$E_A = A(z)E_A(x)e^{-i\beta z},$$
$$E_B = B(z)E_B(x)e^{-i\beta z}$$

$$\begin{bmatrix} A(z) \\ \mathrm{B}(z) \end{bmatrix} = \begin{bmatrix} \cos\left(\frac{\beta_e - \beta_0}{2} z\right) & -j\sin\left(\frac{\beta_e - \beta_0}{2} z\right) \\ -j\sin\left(\frac{\beta_e - \beta_0}{2} z\right) & \cos\left(\frac{\beta_e - \beta_0}{2} z\right) \end{bmatrix} \begin{bmatrix} A(0) \\ B(0) \end{bmatrix}$$



50:50 Splitter

$$\begin{bmatrix} A(L) \\ B(L) \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -j\frac{1}{\sqrt{2}} \\ -j\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} A(0) \\ B(0) \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix} \begin{bmatrix} A(0) \\ B(0) \end{bmatrix}$$

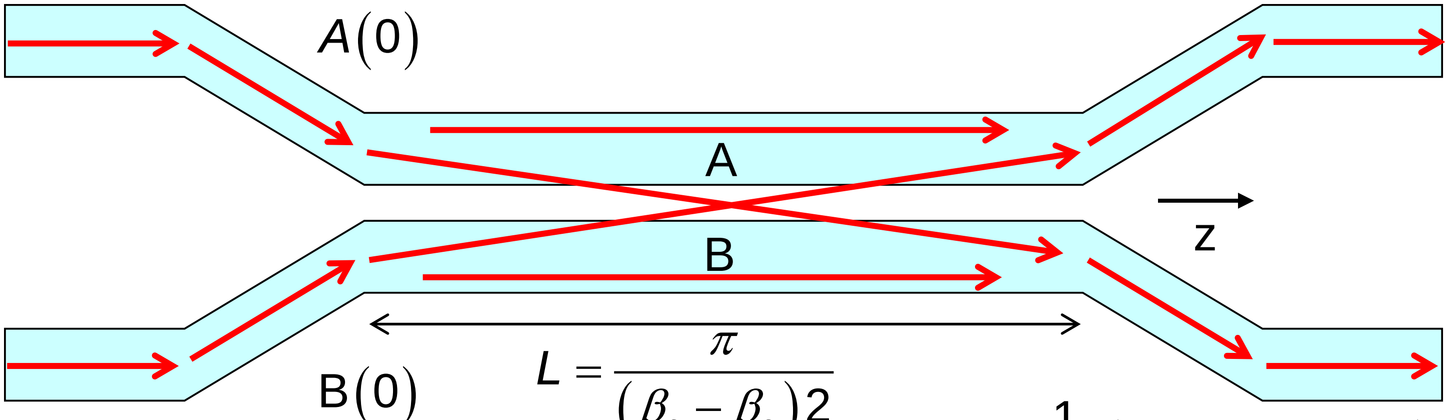


90 Degree Optical Hybrid

Redefining the phase for E at at z=0 and L gives:

$$\begin{bmatrix} A(L) \\ B(L) \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -j\frac{1}{\sqrt{2}} \\ -j\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} A(0) \\ B(0) \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix} \begin{bmatrix} A(0) \\ B(0) \end{bmatrix}$$

$$A(L) = \frac{1}{\sqrt{2}}\left(A(0) - jB(0)\right)$$



$$L = \frac{\pi}{(\beta_e - \beta_o)2}$$

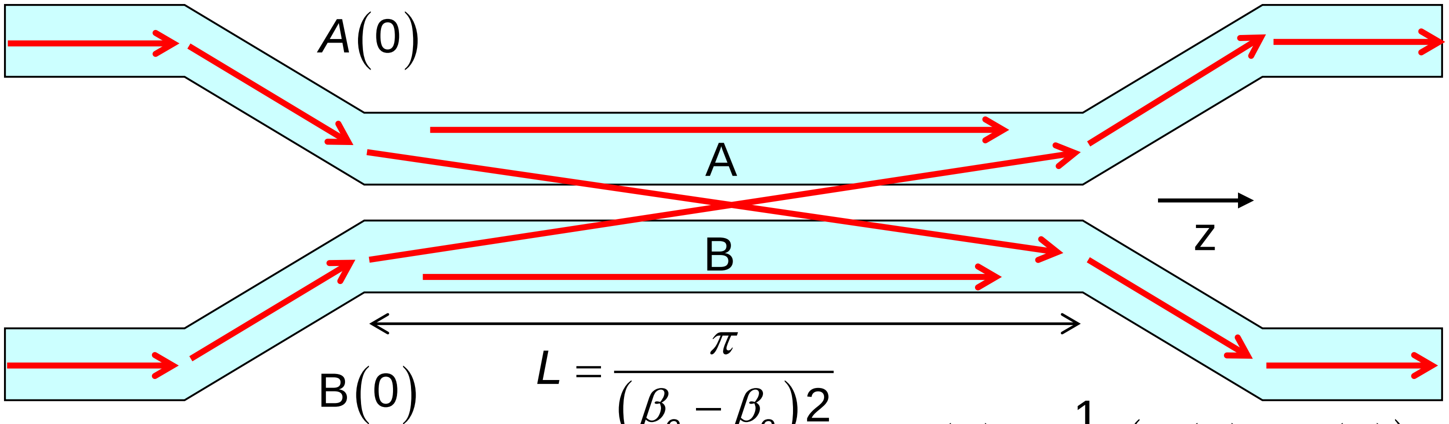
$$B(z) = \frac{1}{\sqrt{2}}\left(-jA(0) - B(0)\right)$$

90 Degree Optical Hybrid

Redefining the phase for input and output arms, we can write:

$$\begin{bmatrix} A(L) \\ \mathrm{B}(L) \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} A(0) \\ B(0) \end{bmatrix}$$

$$A(L) = \frac{1}{\sqrt{2}}\left(A(0) + B(0)\right)$$



$$L = \frac{\pi}{(\beta_e - \beta_o)2}$$

$$\mathrm{B}(z) = \frac{1}{\sqrt{2}}\left(A(0) - B(0)\right)$$