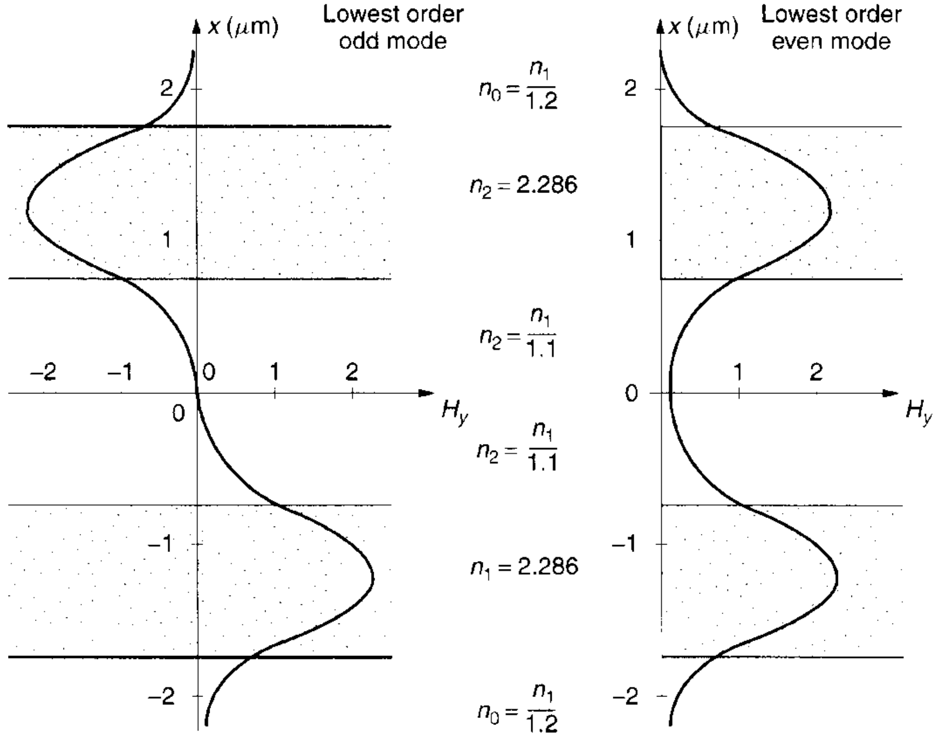
Coupled Waveguides – Super Modes



Elements of Photonics, Iizuka

Figure 9.20 Calculation of the field $H_y(x)$ along the x axis inside the five-layer medium. (Calculation courtesy of R. James.)

$$\beta_o = 10.787\ \mu\text{m}^{-1} \qquad \beta_e = 10.788\ \mu\text{m}^{-1}$$

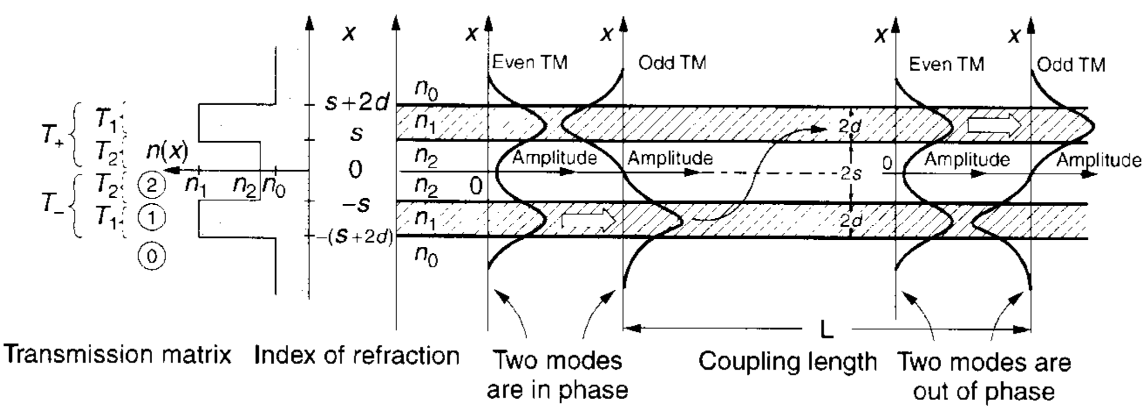


Figure 9.17 Slab optical coupler with symmetry in the refractive index distribution.

$$\boxed{L = \frac{\pi}{\beta_e - \beta_o}}$$ (L = 4.41 mm for the example above)

Coupled Mode Theory – Perturbation Theory

$$\vec{D} = \varepsilon\vec{E} = \varepsilon_0\vec{E} + \vec{P} = \varepsilon_0\vec{E} + \vec{P}_{waveguide} + \vec{P}_{pert}$$

or

$$\vec{D} = \varepsilon\vec{E} + \vec{P}_{pert}$$

The wave equation becomes:

$$\nabla^2\vec{E} - \mu_0\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = \mu_0\frac{\partial^2\vec{P}_{pert}}{\partial t^2}$$

We can write an arbitrary guided wave as:

$$E_{coupled} = A(z)E_A(x)e^{-j(\beta z-\omega t)} + B(z)E_B(x)e^{-j(\beta z-\omega t)}$$

where it is implied that we must eventually take the real part of imaginary quantities.

We plug $E_{coupled}$ into the wave equation (with perturbation) and find:

$$\left(\frac{\partial^2 A}{\partial z^2} - 2j\beta\frac{\partial A}{\partial z}E_A(x) + \frac{\partial^2 B}{\partial z^2} - 2j\beta\frac{\partial B}{\partial z}E_B(x)\right)e^{-j(\beta z-\omega t)}$$

$$= \mu_0\frac{\partial^2 P_{pert}}{\partial t^2}$$

(more detail in appendix below)

A common approximation is the "Slowly Varying Envelope Approximation" - $\left|\frac{\partial^2 A}{\partial z^2}\right|$ and $\left|\frac{\partial^2 B}{\partial z^2}\right| \ll \beta\left|\frac{\partial A}{\partial z}\right|$ and $\beta\left|\frac{\partial B}{\partial z}\right|$ which gives:

$$\left(-2j\beta\frac{\partial A}{\partial z}E_A(x) - 2j\beta\frac{\partial B}{\partial z}E_B(x)\right)e^{-j(\beta z-\omega t)} = \mu_0\frac{\partial^2 P_{pert}}{\partial t^2}$$

Multiplying on the right by $E_A(x)$ or $E_B(x)$ and integrating over x gives the starting equations for coupled mode calculations:

$$\boxed{\frac{\partial A}{\partial z}e^{-j(\beta z-\omega t)} = \frac{j}{2\beta}\frac{\partial^2}{\partial t^2}\int_{-\infty}^{\infty}P_{pert}(x,t)E_A(x)dx}$$

and

$$\boxed{\frac{\partial B}{\partial z}e^{-j(\beta z-\omega t)} = \frac{j}{2\beta}\frac{\partial^2}{\partial t^2}\int_{-\infty}^{\infty}P_{pert}(x,t)E_B(x)dx}$$

where we used the fact that the mode profiles are orthogonal and assumed they are normalized:

$$\int_{-\infty}^{\infty}E_A(x)E_B(x)dx = 0$$

and

$$\int_{-\infty}^{\infty}E_A(x)E_A(x)dx = \int_{-\infty}^{\infty}E_B(x)E_B(x)dx = 1$$

Coupling Between Degenerate Modes in Parallel Slab Waveguides:

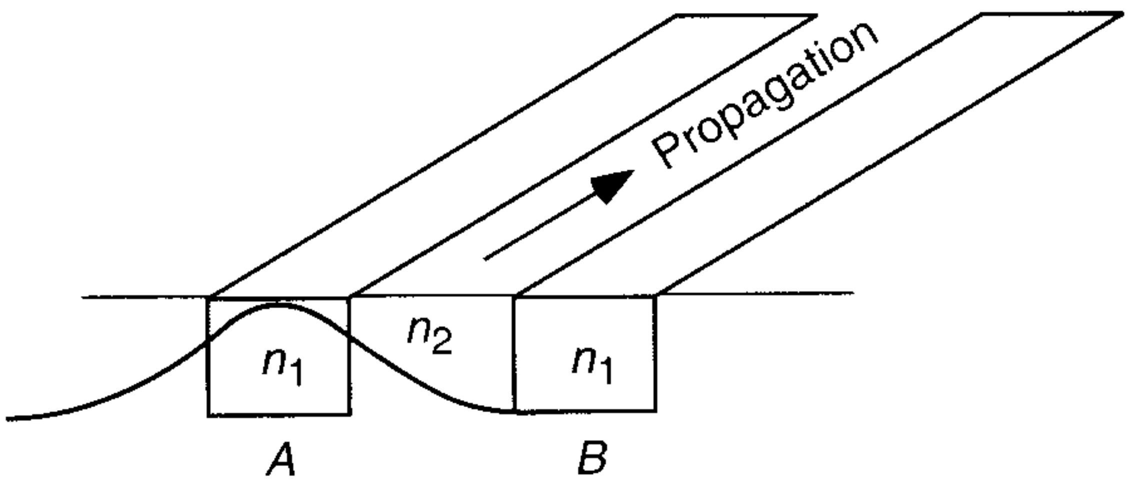


Figure 10.9 A directional coupler can be made by placing two waveguides in close proximity to one another for a finite distance. The evanescent field from one waveguide overlaps the core of the second waveguide, leading to coupling.

From Fundamentals of Optoelectronics, Pollock

The "perturbation" seen by the mode A is due to mode B in the core of B.

$$P_{pert} = \varepsilon_0 \left(n_1^2 - n_2^2\right) B(z) E_B(x) e^{-j(\beta z - \omega t)}$$

Plugging this into the first starting equation for coupled mode calculations gives:

$$\boxed{\frac{\partial A}{\partial z} = -j\kappa B}$$

where the coupling constant is:

$$\kappa = \frac{\omega^2 \varepsilon_0}{2\beta}\left(n_1^2 - n_2^2\right) \int_{\substack{\text{core} \\ \text{of B}}} E_B(x) E_A(x)\, dx$$

similarly

$$\frac{\partial B}{\partial z} = -j\kappa A$$

The equations show that the optical power oscillates between the two slab guides, transferring completely in a length $\pi/2\kappa$.

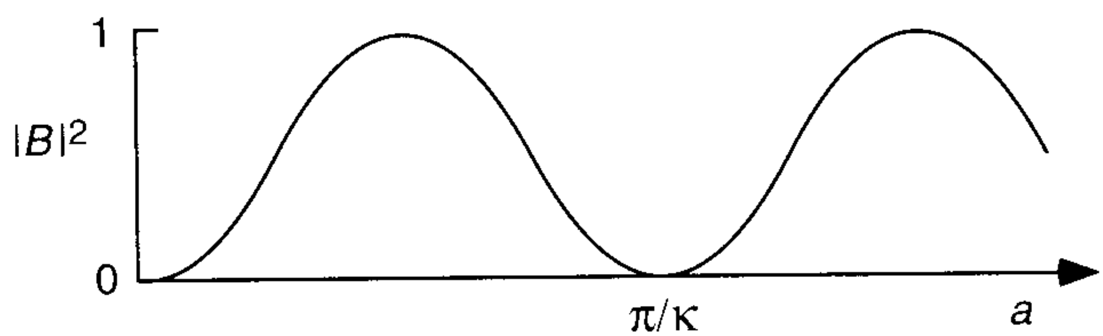


Figure 10.11 The power in waveguide *B* varies sinusoidally as a function of coupling length *a*.

From <u>Fundamentals of Optoelectronics</u>

Connection with Supermode Approach

$$\beta_e - \beta_o = 2\kappa$$

Note: Coupled mode theory can be used to model many other coupled wave phenomena including wave propagation in distributed feedback lasers.

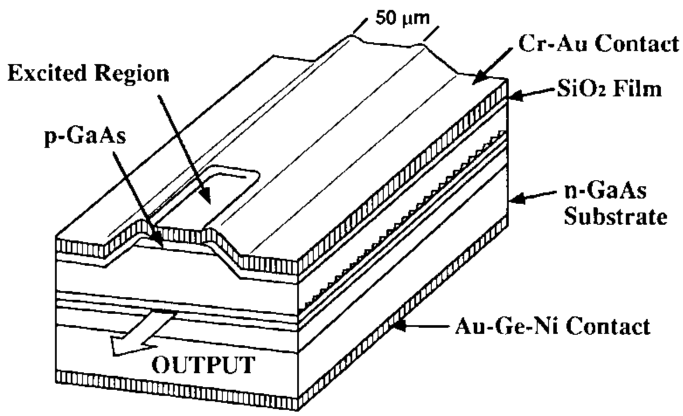


FIGURE 2.1. Schematic structure of a GaAlAs/GaAs distributed feedback laser achieving cw operation at room temperature (after Ref. 6 © 1976 IEEE).

Figure from Semiconductor Lasers: Past, Present, and Future, G. P. Agrawal ed.

Appendix

We start with the driven vector wave equation

$$\nabla^2 \vec{E} - \mu_0 \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \vec{P}_{pert}}{\partial t^2}$$

and a coupled guided wave

$$E_{coupled} = A(z) E_A(x) e^{-j(\beta z - \omega t)} + B(z) E_B(x) e^{-j(\beta z - \omega t)},$$

a scalar quantity that represents the either a TE or TM mode.

Noting that the ortho-normal guided modes $E_A(x)$ and $E_A(x)$ satisfy the homogeneous wave equation

$$\nabla^2 E_{A,B}(x) e^{-j(\beta z - \omega t)} - \mu_0 \varepsilon \frac{\partial^2 E_{A,B}(x) e^{-j(\beta z - \omega t)}}{\partial t^2} = 0.$$

We plug E(x) into the wave equation with perturbation and get

$$\left(\frac{\partial^2 A}{\partial z^2} - 2j\beta \frac{\partial A}{\partial z} E_A(x) + \frac{\partial^2 B}{\partial z^2} - 2j\beta \frac{\partial B}{\partial z} E_B(x) \right) e^{-j(\beta z - \omega t)}$$

$$= \mu_0 \frac{\partial^2 P_{pert}}{\partial t^2}$$

$$\underbrace{\nabla^2 E_A(x)e^{-j(\beta z-\omega t)} - \mu_0\varepsilon\frac{\partial^2 E_A(x)e^{-j(\beta z-\omega t)}}{\partial t^2}}_{0}$$

$$+\underbrace{\nabla^2 E_B(x)e^{-j(\beta z-\omega t)} - \mu_0\varepsilon\frac{\partial^2 E_B(x)e^{-j(\beta z-\omega t)}}{\partial t^2}}_{0}$$

$$+\left[\frac{\partial^2 A}{\partial z^2} - 2j\beta\frac{\partial A}{\partial z}\right]E_A(x)e^{-j(\beta z-\omega t)}$$

$$+\left[\frac{\partial^2 B}{\partial z^2} - 2j\beta\frac{\partial B}{\partial z}\right]E_B(x)e^{-j(\beta z-\omega t)}$$

$$=\mu_0\frac{\partial^2 P_{pert}}{\partial t^2}$$

Then the “Slowly Varying Envelope Approximation” gives:

$$\left(-2j\beta\frac{\partial A}{\partial z}E_A(x) - 2j\beta\frac{\partial B}{\partial z}E_B(x)\right)e^{-j(\beta z-\omega t)} = \mu_0\frac{\partial^2 P_{pert}}{\partial t^2}$$