Coupling Into Waveguides

End-Fire Coupling

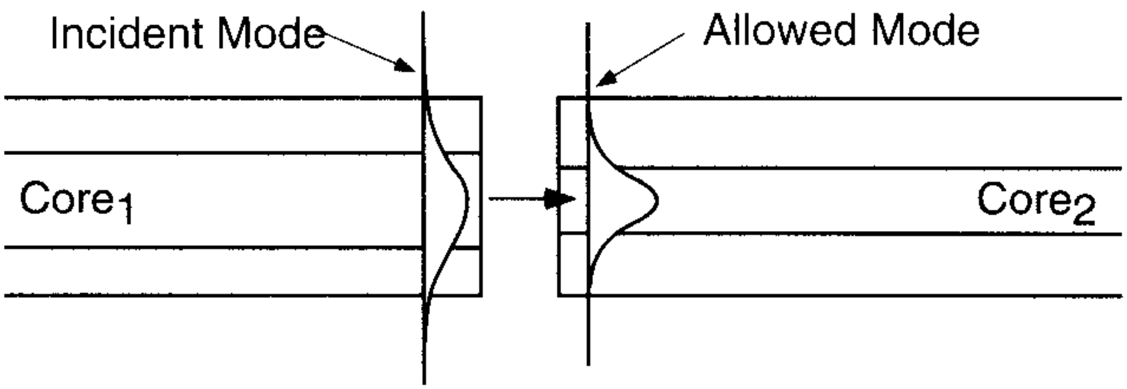


Figure 11.1 An incident mode usually has a different spatial profile than the field in the input waveguide. The overlap of the two modes determines the degree of coupling between the input and guided mode.

Figure 11.1 is from "Fundamentals of Optoelectronics" by Pollock

The continuity of tangential E and H can be used to find reflection and transmission coefficients for the incident mode (see Pollock for the "derivation").

Let n_{eff}^{1} be the effective index for the incident mode and n_{eff}^{2} be the effective index for the transmitted mode.

The reflection coefficients are:

$$r = \frac{n_{eff}^{1} - n_{eff}^{2}}{n_{eff}^{1} + n_{eff}^{2}} \text{ and } R = \left| \frac{n_{eff}^{1} - n_{eff}^{2}}{n_{eff}^{1} + n_{eff}^{2}} \right|^{2}$$

The transmittance is:

$$t = \frac{2\sqrt{n_{eff}^1 n_{eff}^2}}{n_{eff}^1 + n_{eff}^2} \frac{\sqrt{n_{eff}^1 n_{eff}^2}}{2} \sqrt{\frac{\varepsilon_0}{\mu_0}} \int_{-\infty}^{\infty} E_y^1 E_y^{2*} dx$$

for TE modes,

and we infer more generally

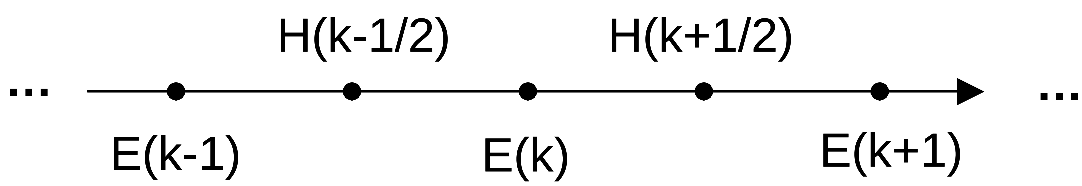
$$T = \frac{4 n_{eff}^1 n_{eff}^2}{\left(n_{eff}^1 + n_{eff}^2\right)^2} \frac{\left[\int E_t^1 E_t^{2*} dA\right]^2}{\left[\int \left|E_t^1\right|^2 dA\right]\left[\int \left|E_t^2\right|^2 dA\right]}$$

where E_t^1 and E_t^2 are the transverse electric fields for the incident and transmitted fields respectively.

Finite Difference Time Domain (FDTD) Simulation for Beam Propagation

<u>One Dimensional FDTD</u>

We solve for E_x and H_y on a one-dimensional grid along z.



We start with initial values and boundary conditions and use the time-dependent, source free Maxwell's equations

$$\boxed{\begin{aligned} \frac{\partial E}{\partial t} &= -\frac{1}{\varepsilon_0}\frac{\partial H}{\partial z} \\ \frac{\partial H}{\partial t} &= -\frac{1}{\mu_0}\frac{\partial E}{\partial z} \end{aligned}}$$

to propagate the field values in time.

The derivatives are found numerically using the "central difference" approximations:

$$\frac{E^{n+1/2}(k)-E^{n-1/2}(k)}{\Delta t} = -\frac{1}{\varepsilon_0}\frac{H^n(k+1/2)-H^n(k-1/2)}{\Delta z}$$

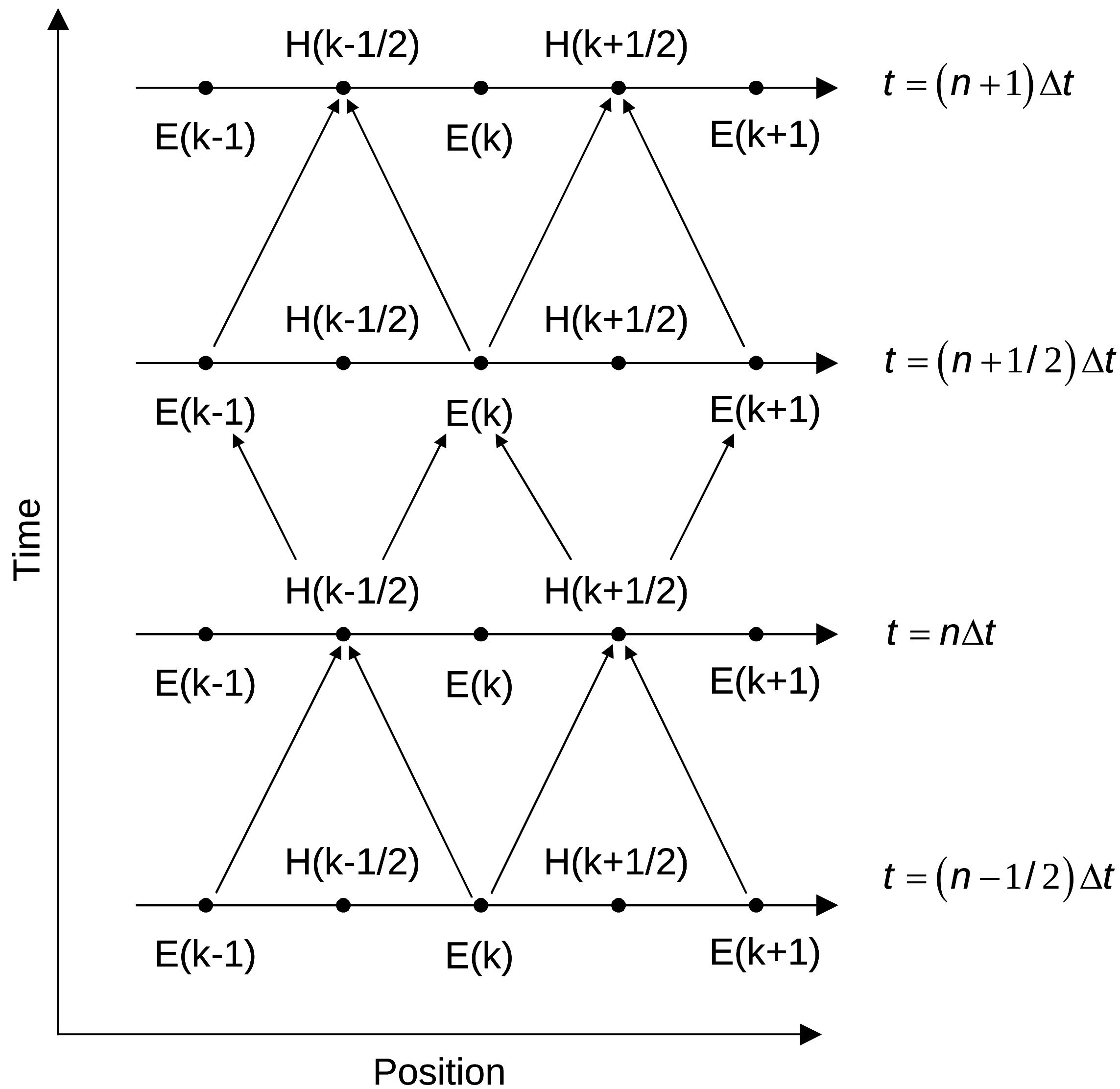
$$\frac{H^{n+1}(k+1/2)-H^n(k+1/2)}{\Delta t} = -\frac{1}{\mu_0}\frac{E^{n+1/2}(k+1)-E^{n+1/2}(k)}{\Delta z}$$

These equations can be re-arranged to give:

$$E^{n+1/2}(k) = E^{n-1/2}(k) - \frac{\Delta t}{\varepsilon_0}\frac{H^n(k+1/2)-H^n(k-1/2)}{\Delta z}$$

$$H^{n+1}(k+1/2) = H^n(k+1/2) - \frac{\Delta t}{\mu_0}\frac{E^{n+1/2}(k+1)-E^{n+1/2}(k)}{\Delta z}$$

This last pair of equations can be represented pictorially.



The Courant stability condition is $c\Delta t \leq \Delta z$. When satisfied, we say that the grid is "causally connected". The justification for this condition goes something like this: The speed of light limits transfer of information in a time Δt to a distance $c\Delta t$. We must take $c\Delta t \leq \Delta z$, to ensure that distant points that can affect the time propagation are included in the numerical scheme.

<u>Three Dimensions</u>

We use a 3-dimensional grid with a "Yee cell".

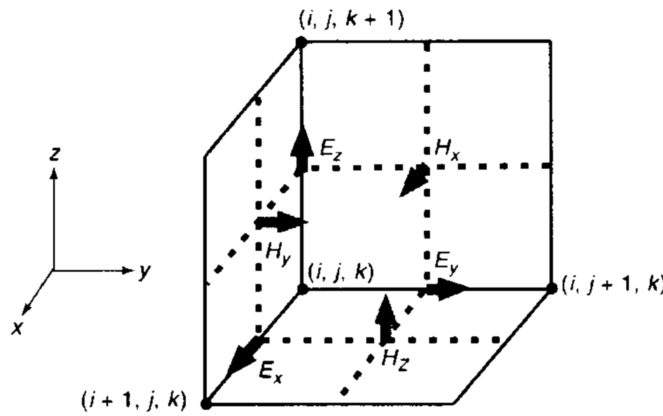


Figure 12.6 Three-dimensional staggered mesh FDTD cell (Yee lattice).

From "Computational Methods for Electromagnetics" by Peterson et al.

FDTD Pros and Cons:

<u>Advantage</u>

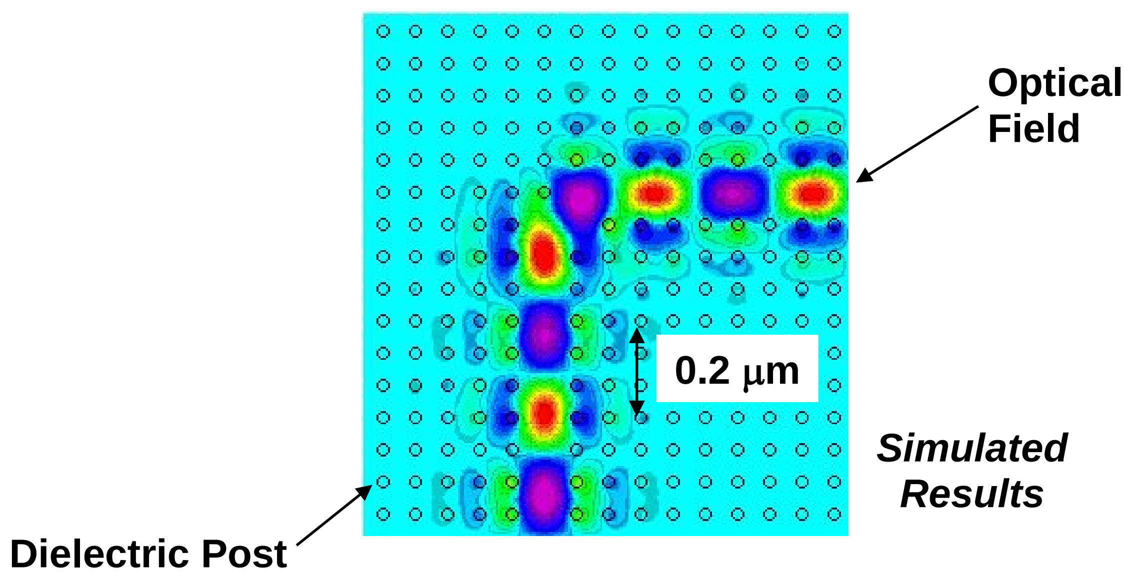
- The method is exact.

<u>Disadvantage</u>

- The method is memory intensive and slow because it stores values for, and operates on, each spatial grid point at each time step.

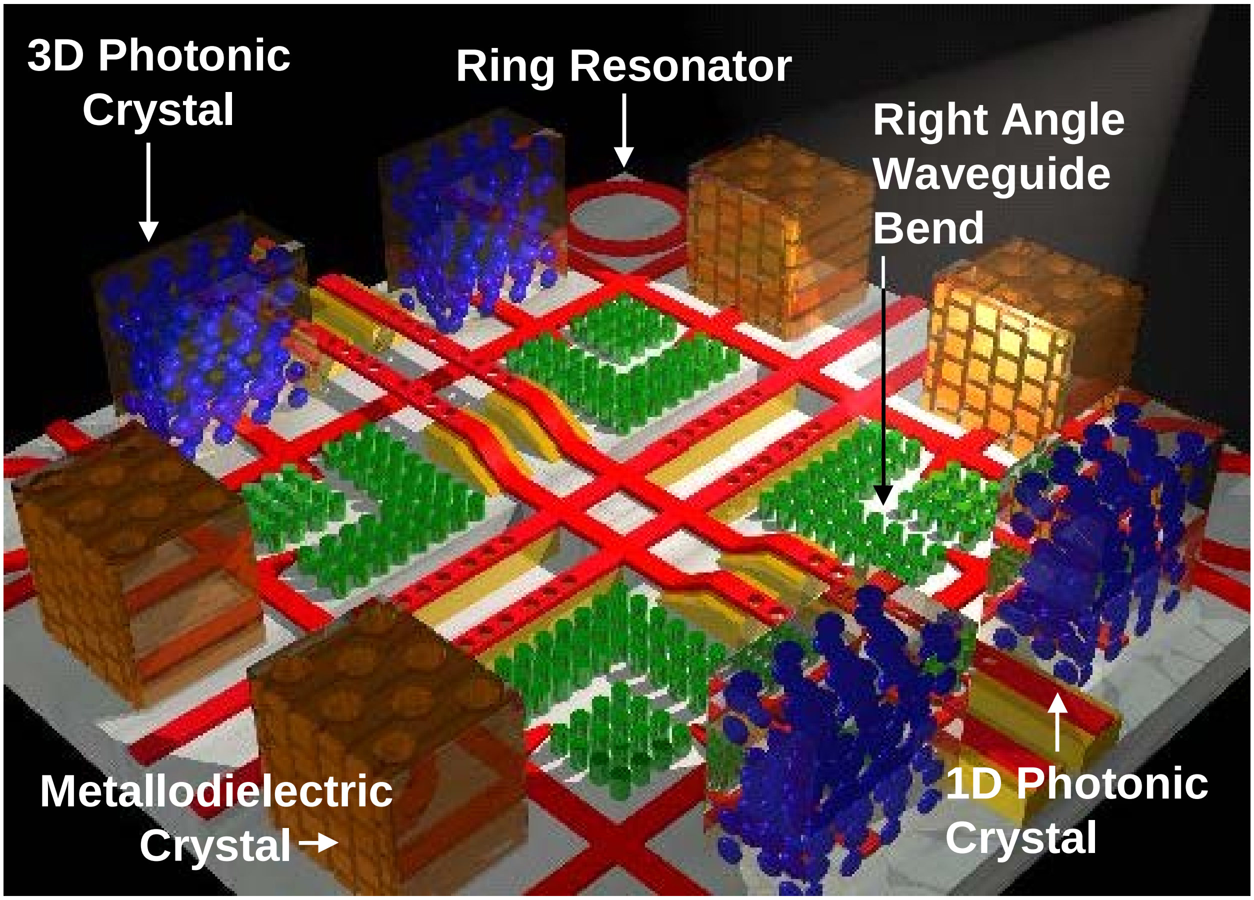
Application: Photonic Crystals

Sharp, right-angle bends are possible in photonic crystals.



From the web page of the Joannopoulos Research Group at MIT

The Photonic Micropolis



From the web page of the Joannopoulos Research Group at MIT

The Beam Propagation Method

We start with the Helmholtz equation for components of the electric field:

$$\nabla^2 E(\vec{r}) + n^2 k^2 E(\vec{r}) = 0$$

then write $E(\vec{r})$ as

$$E(\vec{r}) = E(x,y,z) = U(x,y,z)e^{-jn_{clad}k_0 z}$$

What we are doing here is separating $E(\vec{r})$ into a two parts: a slowly varying envelope factor $U(x,y,z)$ and a rapidly varying phase factor $e^{-jn_{clad}k_0 z}$. We are implicitly assuming that we have a wave that propagates primarily along z (the Paraxial Wave Approximation) with a slowly varying x-y profile and an amplitude that varies slowly along z.

Inserting $U(x,y,z)e^{-jn_{clad}k_0 z}$ into the Helmholtz equation gives

$$\frac{\partial^2 U}{\partial z^2} - 2jn_{clad}k_0 \frac{\partial U}{\partial z} + \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \left(n^2 - n_{clad}{}^2\right)k_0^2 U = 0$$

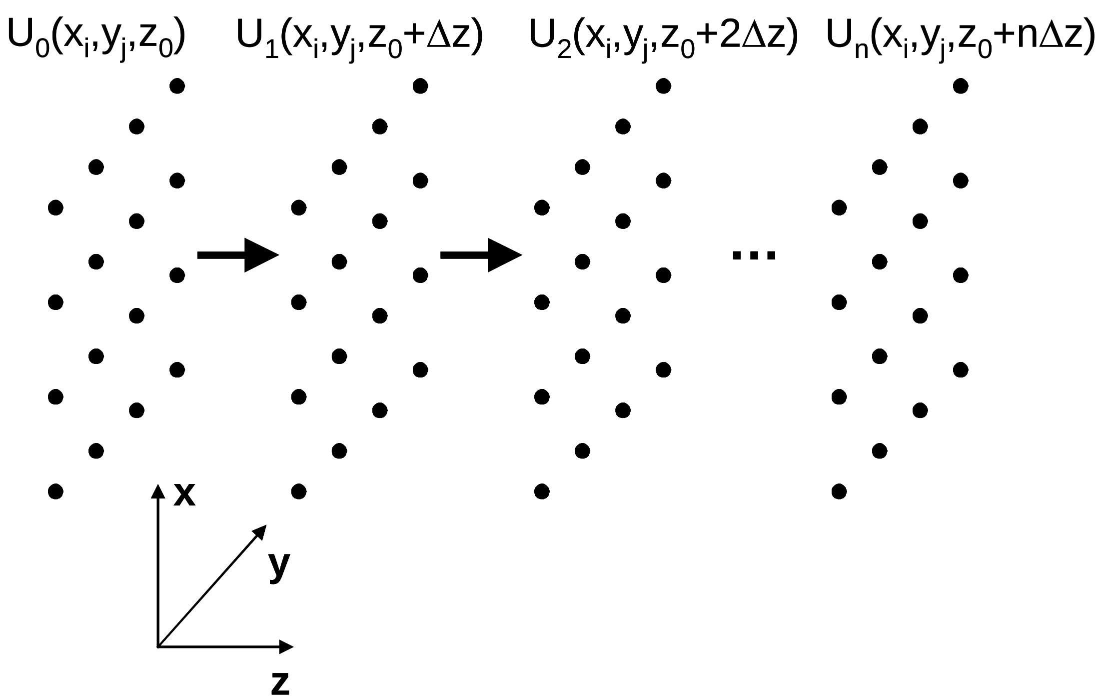
Next we use the Slowly Varying Envelope Approximation

$$\left|\frac{\partial^2 U}{\partial z^2}\right| \ll \left|2n_{clad}k_0 \frac{\partial U}{\partial z}\right|$$

to get the basic BPM equation.

$$\boxed{\frac{\partial U}{\partial z} = -\frac{j}{2n_{clad}k_0}\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right) - \frac{j}{2n_{clad}}\left(n^2 - n_{clad}{}^2\right)k_0 U}$$

To solve this "initial value" problem we specify U(x,y,z) at a plane $z = z_0$. Then we iterate U along the z-axis using finite differences for the x and y derivatives (FD-BPM). The numerical solution of this equation can be represented pictorially.



BPM Pros and Cons:

What we have gained:

- Speed and reduced computer memory requirements.

What we have given up:

- The ability to handle reflections and beams propagating at a large angle to the z-axis.