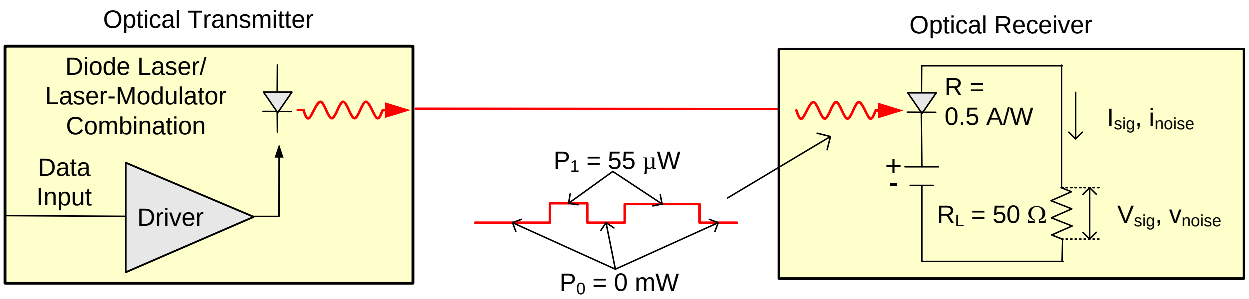
OPTI 500D, Take Home Exam

Signal, Noise, Bit Error Rate, Signal-to-Noise Ratio, Q-Factor

You may use class notes and other reference material.

Due February 29, 2012



An optical data link includes a transmitter, and optical fiber, and an optical receiver, as shown above. The transmitter sends a series of logical ones and zeros by turning the power on and off for a semiconductor laser/optical modulator combination. The time-averaged power received at the photodiode is 0 mW for a logical zero and 55 μW for a logical one. We neglect impairments due chromatic dispersion and polarization mode dispersion. The receiver includes a photodiode with responsivity R = 0.5 A/W that is connected to a load resistor R_L = 50 ohms. The data rate is 10 Gbits/sec and the receiver has a bandwidth Δf = 15.5 Ghz. The receiver is at room temperature T = 300 K.

The signal current through the resistor in the receiver can be calculated from the optical power using the responsivity R of the photodiode. The signal current is composed of a DC component equal to the mean of the signal current averaged over a bit period and a time-varying noise component.

Signal

1. Calculate the time-averaged (DC) currents I_1 and I_0 that correspond to the logical "ones" and "zeroes" that arrive in the optical signal at the optical receiver.

Noise

The shot noise current at the load resistor in the receiver is the statistical variance in the signal current due to the shot process and is given by the expression

$$i_{shot}^2 = \sigma_s^2 = 2qI\Delta f \, ,$$

where q is the electronic charge, I is the time-averaged signal current, and we have neglected the contributions due to the small photodiode dark current I_d.

2. Find the shot noise currents $\sigma_{s,1}$ and $\sigma_{s,0}$ that correspond to the currents I_1 and I_0.

The noise current at the load resistor due to thermal noise in the receiver is the statistical variance in the signal current due to thermal processes. It does not depend on the time-averaged signal current is given by the expression

$$i_{therm}^2 = \sigma_T^2 = \frac{4k_B T}{R_L}\Delta f$$

3. Find the thermal noise current σ_T.

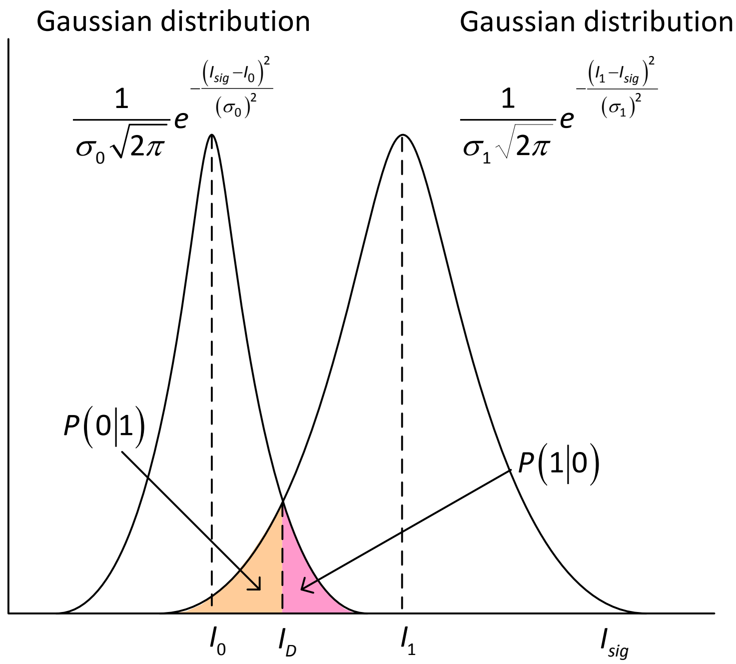
The total noise current at the load resistor is determined by noting that the variances of two independent probability distributions sum to give a total variance. This means that

$$i_{noise}^2 = \sigma_{tot}^2 = \sigma_s^2 + \sigma_T^2 = i_{shot}^2 + i_{therm}^2 = 2qI\Delta f + \frac{4k_B T}{R_L}\Delta f$$

4. Find the total noise currents σ_1^2 and σ_0^2 that correspond to the currents I_1 and I_0.

Bit Error Rate

The bit error rate (BER) is the probability that a transmitted bit will be incorrectly read at the receiver. Incorrectly read means that a zero is detected as a one or vice versa. To relate the bit error rate to the signal currents and the noise currents, we start by considering the figure below.



We assume that the signal currents for zeros and ones have Gaussian distributions with means of I_0 and I_1, respectively, and standard deviations of σ_0 and σ_1, respectively. Note that what we actually measure is signal voltage $V_{sig} = I_{sig}R_L$ and noise voltage $v_{noise} = \sigma_{tot}R_L$. The threshold (decision) current I_D is the reference current that is used to distinguish a zero from a one. If measurement indicates the current is less than I_D, we detect a zero. On the other hand, if measurement indicates the current is greater than I_D, we detect a one. It turns out that the optimum threshold current is very close to

$$I_D = \frac{\sigma_0 I_1 + \sigma_1 I_0}{\sigma_0 + \sigma_1},$$

which is the current for which the two Gaussian curves cross.

There are two types of transmission errors and a different probability for each. $P(0|1)$ is the probability that a zero is detected when a one was transmitted. $P(1|0)$ is the probability that a one is detected when a zero was transmitted. These probabilities are given by

$$P(0|1) = \frac{1}{\sigma_1\sqrt{2\pi}} \int_{-\infty}^{I_D} e^{-\left(I_{sig}-I_1\right)^2 / 2(\sigma_1)^2} dV_{signal} = \frac{1}{2} erfc\left(\frac{I_1 - I_D}{\sigma_1}\right)$$

and

$$P(1|0) = \frac{1}{\sigma_o\sqrt{2\pi}} \int_{-\infty}^{I_D} e^{-\left(I_{sig}-I_0\right)^2 / 2(\sigma_0)^2} dV_{signal} = \frac{1}{2} erfc\left(\frac{I_D - I_0}{\sigma_0}\right)$$

The bit error rate is the total probability of an error:

$$\begin{aligned} BER = P(\text{error}) &= P(0|1)P(1) + P(1|0)P(0) = \frac{1}{2}\left[P(0|1) + P(1|0)\right] \\ &= \frac{1}{4} erfc\left(\frac{I_1 - I_D}{\sigma_1\sqrt{2}}\right) + \frac{1}{4} erfc\left(\frac{I_D - I_0}{\sigma_0\sqrt{2}}\right) \end{aligned},$$

where erfc is the complementary error function and we have assumed that there are equal numbers of zeros and ones in the data stream so that $P(0) = P(1) = \frac{1}{2}$. A bit rate of 10^{-6} means that, on average, one detection error is made in the transmission of 1,000,000 bits.

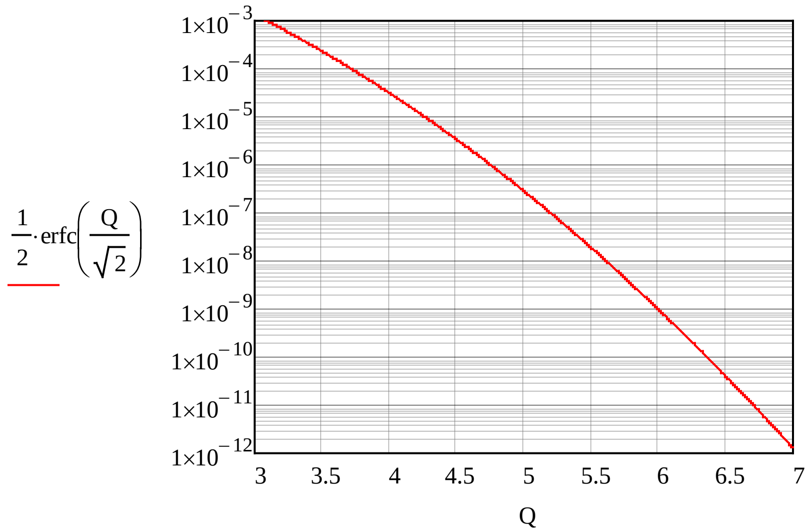
Signal-to-Noise Ratio

The signal to noise ratio for zeros and ones are given by the ratio of the average signal power divided by the noise power:

$$SNR_1 = \frac{I_1^2}{\sigma_1^2} \text{ and } SNR_0 = \frac{I_0^2}{\sigma_0^2}\ .$$

5. Find the signal to noise ratios SNR_1 and SNR_0.

Examining the expressions for the signal-to-noise ratios and the bit error rate, we see that the bit error rate cannot be simply expressed in terms of the signal to noise ratios. This is where the Q-factor comes in.



Q-Factor

6. Show that $\frac{I_1 - I_D}{\sigma_1} = \frac{I_D - I_0}{\sigma_0} = \frac{I_1 - I_0}{\sigma_0 + \sigma_1}$, when the threshold current I_D has the nearly optimum value given above.

If we define $Q \equiv \frac{I_1 - I_0}{\sigma_0 + \sigma_1}$, the expression for the bit error rate can be written

$$BER = \frac{1}{2} erfc\left(\frac{Q}{\sqrt{2}}\right).$$

What we have gained by defining Q, is to create a quantity that looks like a signal-to-noise ratio that can be used in a simple expression for the bit error rate, which is displayed in the graph above.

7. Find Q and then use the graph to find the bit error rate for the transmission link.