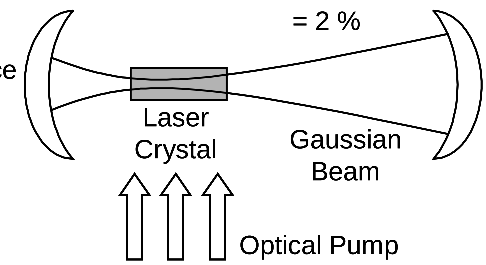
OPTI 500 D, Spring 2012, Homework #1
Solutions

1.



The optical loss inside the cavity of a solid state laser, illustrated above, is 2 % per pass due to light scattering in the laser crystal. The optical beam in the laser has a spot size much smaller than the cross-section of the crystal. The gain coefficient in the crystal is 0.13 cm^{-1}. Find the minimum length of the crystal for lasing. (10 points)

The balance equation for a laser is:

$$e^{2g\ell} R_1 R_2 (1-L_i)^2 = 1$$

$$\Rightarrow \ell = \frac{\ln\left(\frac{1}{R_1 R_2 (1-L_i)^2}\right)}{2g} = \frac{\ln\left(\frac{1}{0.99 \cdot 0.90(1-0.02)^2}\right)}{2 \cdot 0.13}$$

$$\Rightarrow \boxed{\ell = 6.0\ \text{mm}}$$

2. In the absence of an applied electric field, GaAs is isotropic and the index ellipsoid is given by:

$$\frac{x^2}{n^2}+\frac{y^2}{n^2}+\frac{z^2}{n^2}=1,$$

where x, y, and z coincide with crystal axes. The r matrix for GaAs is:

$$\vec{r}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\\r_{41}&0&0\\0&r_{41}&0\\0&0&r_{41}\end{bmatrix}.$$

a. Sketch the projection of the index ellipsoid in the x-y plane for GaAs when an electric field is applied in the z-direction. (10 points)

The only non-zero term in the vector of index changes is

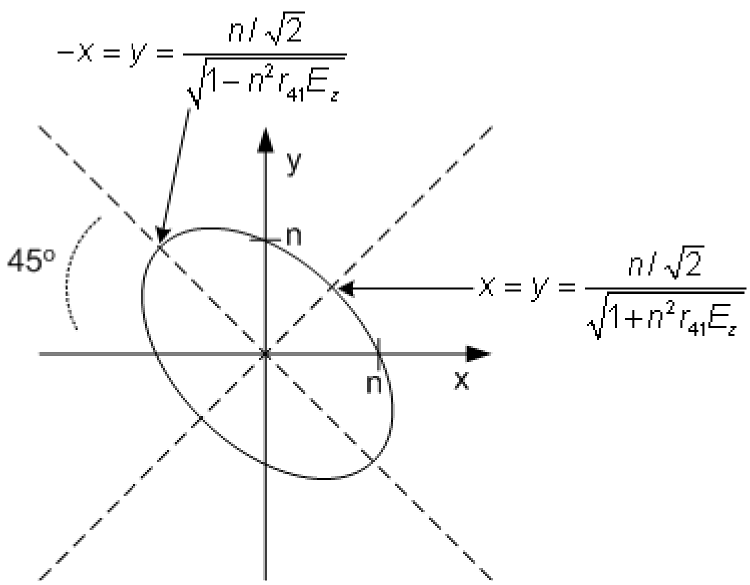
$$\Delta\frac{1}{n_{xy}}=r_{41}E_z$$

so the index ellipsoid in the applied field is given by:

$$\frac{x^2}{n^2}+\frac{y^2}{n^2}+\frac{z^2}{n^2}+r_{41}E_z 2xy=1$$

which, in the x-y plane, reduces to

$$\frac{x^2}{n^2}+\frac{y^2}{n^2}+r_{41}E_z 2xy=1.$$



b. Show that a new set of axes x’, y’, and z’ defined by:

$$x = x'\cos\left(45^\circ\right) + y'\sin\left(45^\circ\right)$$

$$y = -x'\sin\left(45^\circ\right) + y'\cos\left(45^\circ\right)$$

$$z = z'$$

are the principal axes for GaAs with an electric field applied along the z-axis. (10 points)

$\cos\left(45^\circ\right) = \sin\left(45^\circ\right) = \dfrac{1}{\sqrt{2}}$, so

$x = \dfrac{x'}{\sqrt{2}} + \dfrac{y'}{\sqrt{2}}$ and $y = \dfrac{-x'}{\sqrt{2}} + \dfrac{y'}{\sqrt{2}}$

$\Rightarrow x^2 = \dfrac{x'^2}{2} + x'y' + \dfrac{y'^2}{2}$ and $y^2 = \dfrac{x'^2}{2} - x'y' + \dfrac{y'^2}{2}$ and $xy = \dfrac{-x'^2}{2} + \dfrac{y'^2}{2}$

Substituting the above expressions for x, y, and z into

$$\frac{x^2}{n^2} + \frac{y^2}{n^2} + \frac{z^2}{n^2} + r_{41}E_z 2xy = 1$$

gives

$$\frac{\dfrac{x'^2}{2} + x'y' + \dfrac{y'^2}{2}}{n^2} + \frac{\dfrac{x'^2}{2} - x'y' + \dfrac{y'^2}{2}}{n^2} + \frac{z'^2}{n^2} + r_{41}E_z 2x'^2\left(-\frac{x'^2}{2} + \frac{y'^2}{2}\right) = 1$$

which reduces to

$$\frac{x'^2\left(1 - r_{41}E_z n^2\right)}{n^2} + \frac{y'^2\left(1 + r_{41}E_z n^2\right)}{n^2} + \frac{z'^2}{n^2} = 1.$$

The absence of cross-terms indicates that x’, y’, and z’ are the principal axes.