OPTI 500 D, Spring 2012, Take Home Exam Solutions

First some constants

$q := 1.602\cdot 10^{-19}\,C$ $\Delta f := 15.5\cdot 10^{9}\,Hz$ $R := 0.5\cdot\frac{A}{W}$ $P1 := .000055\cdot W$ $P0 := 0\cdot W$

$T := 300\cdot K$ $kB := 1.3806503\cdot 10^{-23}\cdot\frac{m^2\cdot kg}{s^2\cdot K}$ $RL := 50\cdot ohm$

Signal Currents

$I1 := P1\cdot R = 2.75\times 10^{-5}\,A$ $I0 := P0\cdot R = 0$

Shot Noise Currents

$\sigma s1 := \sqrt{2\cdot q\cdot I1\cdot \Delta f} = 3.696\times 10^{-7}\,A$ $\sigma s0 := \sqrt{2\cdot q\cdot I0\cdot\Delta f} = 0$

Thermal Noise Current

$$\sigma T := \sqrt{\frac{4\cdot kB\cdot T}{RL}\cdot\Delta f} = 2.266\times 10^{-6}\,A$$

Total Noise Currents

$\sigma 1 := \sqrt{\sigma s1^2 + \sigma T^2} = 2.296\times 10^{-6}\,A$ $\sigma 0 := \sqrt{\sigma s0^2 + \sigma T^2} = 2.266\times 10^{-6}\,A$

Signal-to-Noise Ratios

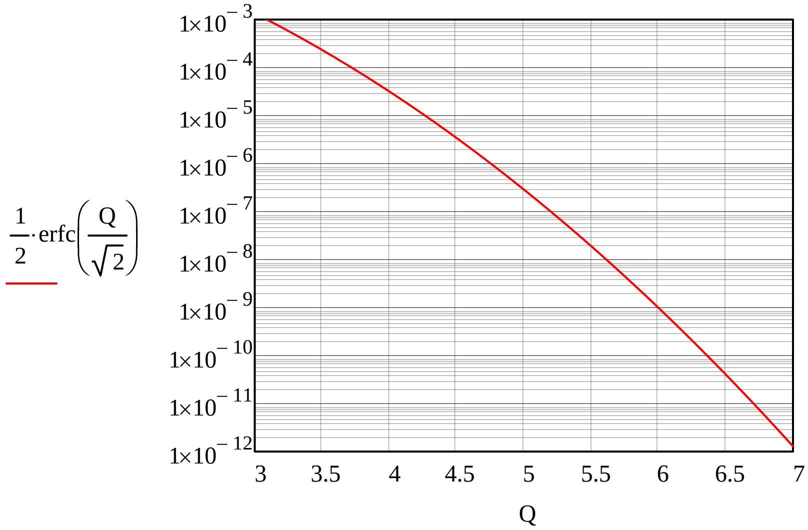
$SNR1 := \frac{I1^2}{\sigma 1^2} = 143.43$ $SNR0 := \frac{I0^2}{\sigma 0^2} = 0$

Proof that $\frac{I_1 - I_D}{\sigma_1} = \frac{I_D - I_0}{\sigma_0} = \frac{I_1 - I_0}{\sigma_0 + \sigma_1}$

$$\frac{I_1 - I_D}{\sigma_1} = \frac{I_1 - \frac{\sigma_0 I_1 + \sigma_1 I_0}{\sigma_0+\sigma_1}}{\sigma_1} = \frac{\cancel{\sigma_0 I_1} + \sigma_1 I_1 - \cancel{\sigma_0 I_1} - \sigma_1 I_0}{\sigma_1(\sigma_0+\sigma_1)} = \frac{\sigma_1 I_1 - \sigma_1 I_0}{\sigma_1(\sigma_0+\sigma_1)} = \frac{\sigma_1(I_1 - I_0)}{\sigma_1(\sigma_0+\sigma_1)} = \frac{I_1 - I_0}{\sigma_0+\sigma_1}$$

$$\frac{I_D - I_0}{\sigma_0} = \frac{\frac{\sigma_0 I_1 + \sigma_1 I_0}{\sigma_0+\sigma_1} - I_0}{\sigma_0} = \frac{\sigma_0 I_1 + \cancel{\sigma_1 I_0} - \sigma_0 I_0 - \cancel{\sigma_1 I_0}}{\sigma_0(\sigma_0+\sigma_1)} = \frac{\sigma_0 I_1 - \sigma_0 I_0}{\sigma_0(\sigma_0+\sigma_1)} = \frac{\sigma_0(I_1 - I_0)}{\sigma_0(\sigma_0+\sigma_1)} = \frac{I_1 - I_0}{\sigma_0+\sigma_1}$$

Q-Factor



$$Q := \frac{I1 - I0}{\sigma 0 + \sigma 1} = 6.027$$

This Q-Factor corresponds to a BER of 10^-9