Linear Optics

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{P} = \varepsilon_0 \chi \vec{E}$$

A linear relationship between E and the polarization P, means that the susceptibility χ is a scalar and a constant.

Linear Optics

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{P} = \varepsilon_0 \chi \vec{E}$$

$$\Rightarrow \vec{D} = \varepsilon_0 \vec{E} + \varepsilon_0 \chi \vec{E} = \underbrace{\varepsilon_0 \left(1 + \chi\right)}_{\varepsilon} \vec{E}$$

A linear relationship between E and the polarization P, means that the dielectric constant ε is a scalar and a constant.

Linear Optics

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

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$$\Rightarrow \vec{D} = \varepsilon_0 \vec{E} + \varepsilon_0 \chi \vec{E} = \underbrace{\varepsilon_0 \left(1 + \chi\right)}_{\varepsilon} \vec{E}$$

A linear relationship between E and the polarization P, means that the dielectric constant ε is a scalar and a constant.

Linear Optics

$$\varepsilon = \varepsilon_1 - i\varepsilon_2$$

$$n = \tfrac{1}{2}\sqrt{\sqrt{\varepsilon_1^{\ 2} + \varepsilon_1^{\ 2}} + \varepsilon_1}$$

$$\alpha = \tfrac{4\pi}{\lambda}\tfrac{1}{2}\sqrt{\sqrt{\varepsilon_1^{\ 2} + \varepsilon_1^{\ 2}} - \varepsilon_1}$$

The constant dielectric and is complex and we can describe wave propagation in the medium with a refractive index n and an absorption coefficient α.

Nonlinear Optics

$$P = P\left(E\right) = P^1 + \frac{1}{2!}\frac{dP}{dE}E + \frac{1}{3!}\frac{d^2P}{d^2E}E^2 + \cdots$$

$$= \chi^1 E + \chi^2 E^2 + \chi^3 E^3 + \cdots$$

More generally, P may be a nonlinear function of E which can be expressed as a Taylor series.

Kerr Effect in Isotropic Crystals

$$P = \chi^1 E + \underbrace{\chi^2 E^2}_{0} + \chi^3 E^3 + \underbrace{\chi^4 E^4 + \cdots}_{\text{small}}$$

$$\Rightarrow P = \chi^1 E + \chi^3 E^3$$

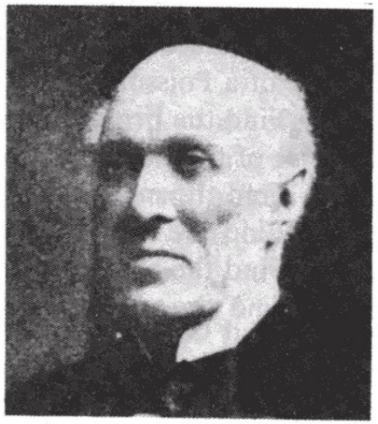
$$\Rightarrow \varepsilon = \varepsilon_0 \left(1 + \chi^1 + \chi^3 \underbrace{E^2}_{\alpha I} \right)$$

In isotropic crystals $\chi^2 = 0$ and terms higher than the χ^3 term are small.

Kerr Effect in Isotropic Crystals

$$\varepsilon = \varepsilon_0 \left(1 + \chi^1 + \chi^3 \underbrace{E^2}_{\alpha I} \right)$$

$$\Rightarrow n = n_0 + n_2 I$$



John Kerr (1824–1907) discovered the quadratic electro-optic effect in 1875.

If the χ^3 term is real, it gives an intensity dependent refractive index (but no change in the absorption coefficient)

Self-Phase Modulation

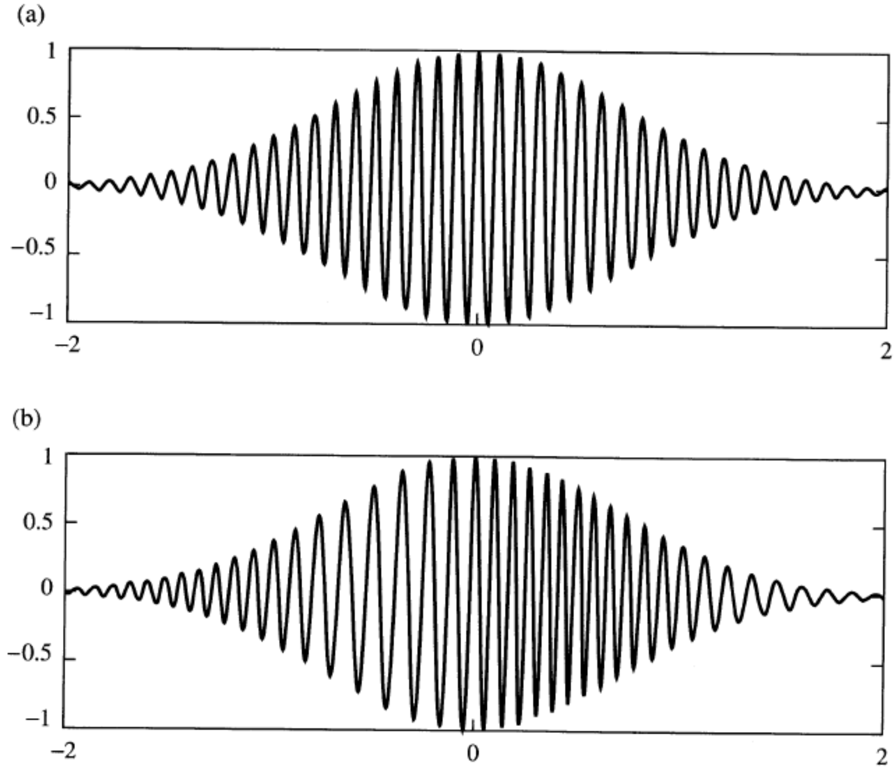


Figure 14.1 Electric field as a function of time: (a) at the input end of the fiber and (b) at the output end of the fiber. The parameters used are $A_{\text{eff}} = 100\ \mu\text{m}^2$, $P = 5$ mW, $L = 2500$ km, $\tau = 0.05$ ps, and $n_2 = 3 \times 10^{-20}\ \text{m}^2/\text{W}$.

$$n = n_0 + n_2 I$$

$$I(t) = I_0 \exp\left(-t^2/\tau^2\right)$$

$$\Delta\omega = \omega(t) - \omega_0$$

$$= \frac{2\pi}{\lambda_0} n_2 L \frac{2t}{\tau^2} I_0 \exp\left(-t^2/\tau^2\right)$$

From *Photonics*,
Yariv and Yeh, 4th Edition

- An increasing intensity, and refractive index, has the effect of decreasing frequency (increasing wavelength) and vice versa

Chromatic Dispersion as an Impairment in Optical Communication Links

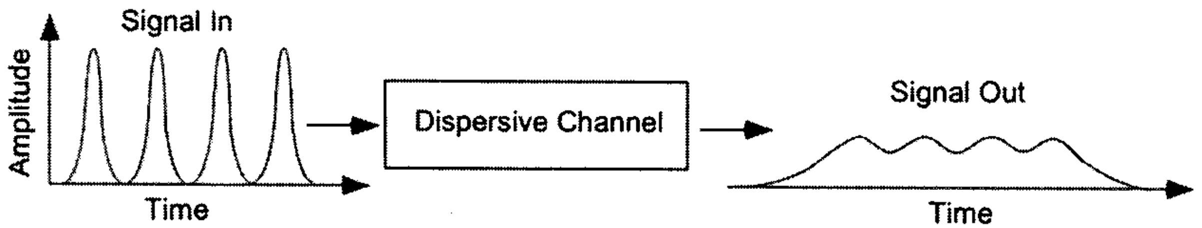


Figure 6.1. Short temporal pulses experience pulse spreading in a dispersive media. If they are not separated by enough time they will begin to overlap.

From Optoelectronics, S. O. Kasap, (Prentice Hall, 1999)

- Chromatic dispersion leads to optical pulse spreading in optical fibers, which can lead to loss of information.

First Report of a Soliton

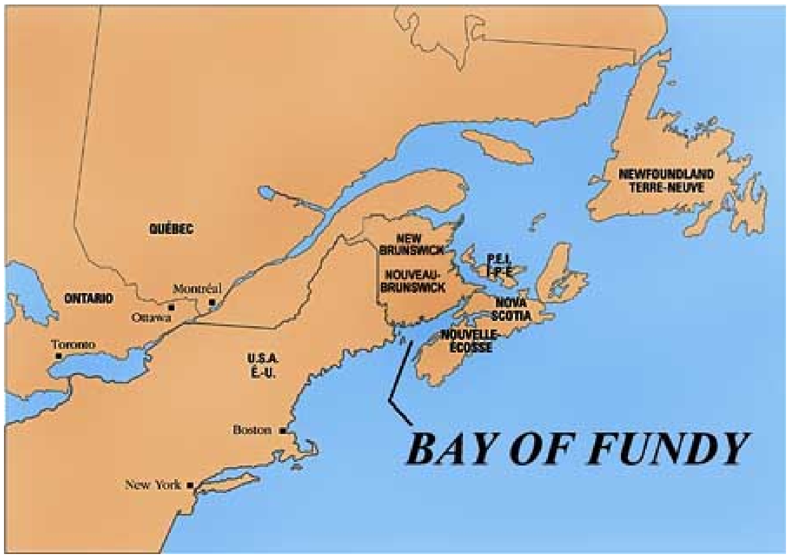
I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped - not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation

This is a most beautiful and extraordinary phenomenon: the first day I saw it was the happiest day of my life. Nobody had ever had the good fortune to see it before or, at all events, to know what it meant. It is now known as the solitary wave of translation.

John Scott Russell describing his observation of a soliton in the Union Canal near Herriot Watt University in Scotland

From “Solitons in Mathematics and Physics,” Alan C. Newell (Society for Industrial and Applied Mathematics, 1985)

Tidal Bore off the Bay of Fundy

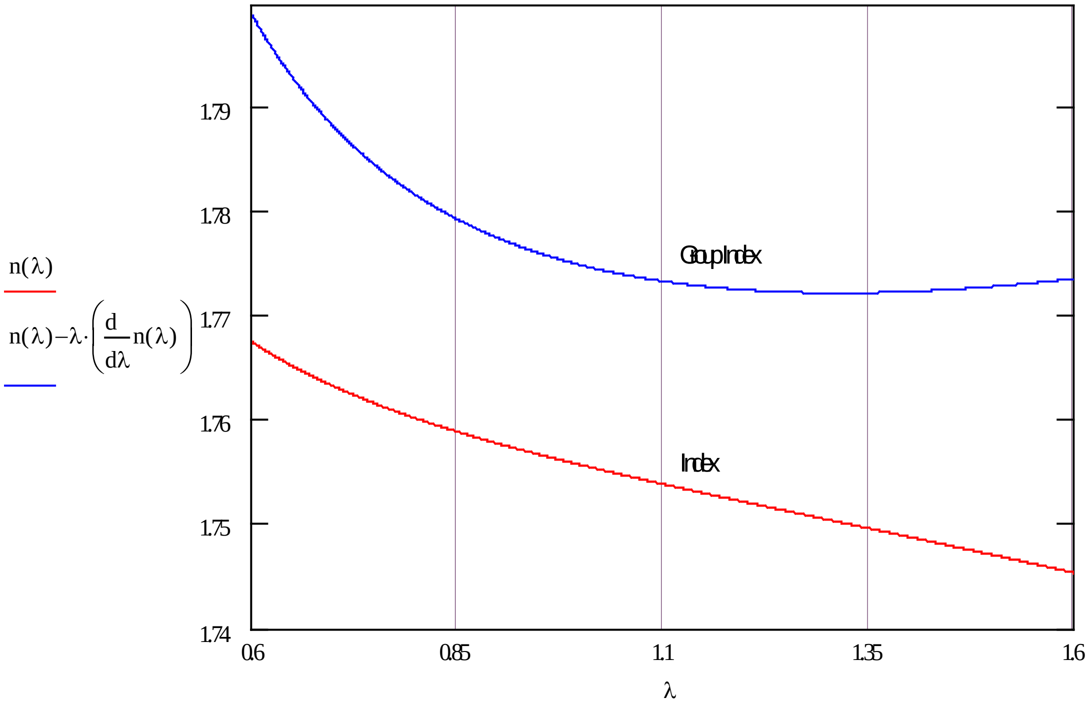


- The Bay of Fundy has the worlds highest tides ~ 20 meters.
- A rising tide in the Bay of Funday (Nova Scotia, Canada) funnels water into inlets and generates tidal bores (water soliton)

Mascaret (Bore) Bordeuax, France

Qian Tang River Bore – Hang Zhou, China, 2004

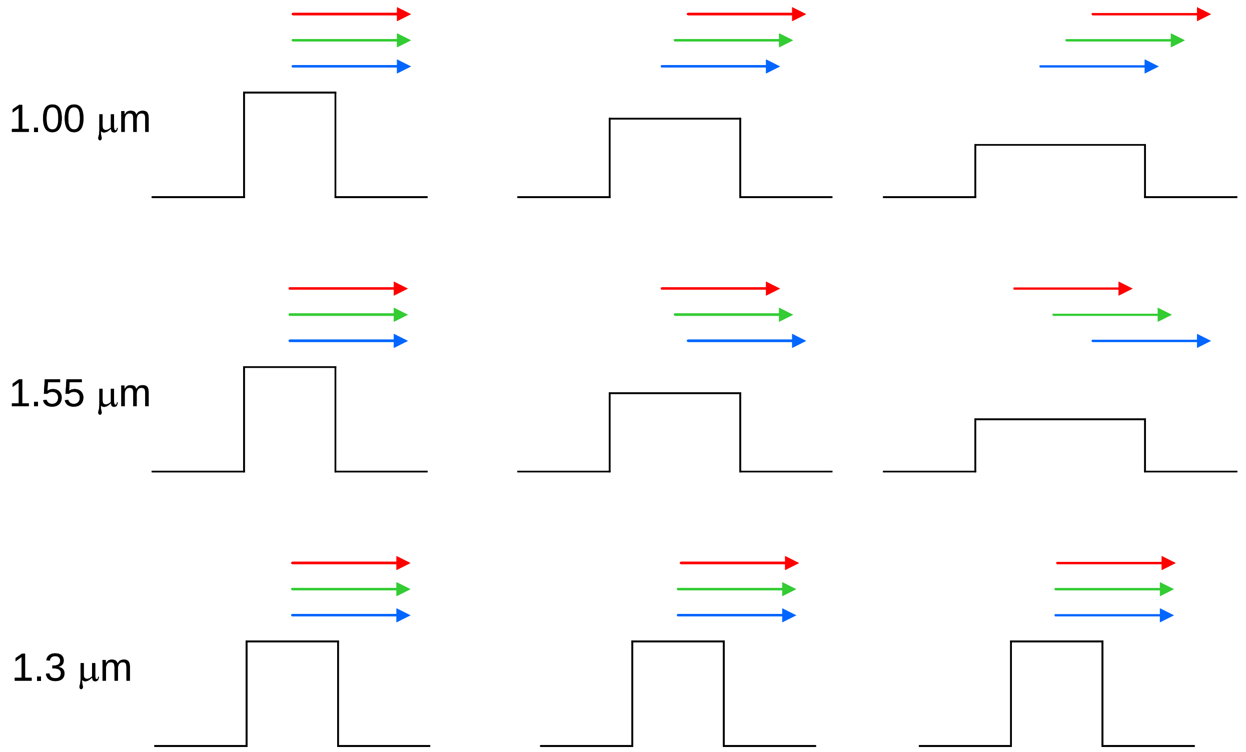
Phase and Group Velocity in Optical Fibers



$$\text{Phase Velocity} \equiv \frac{\omega}{k} = \frac{c}{n}$$

$$\text{Group Velocity:} \quad v_g \equiv \frac{d\omega}{dk} = \frac{c}{n_g}$$

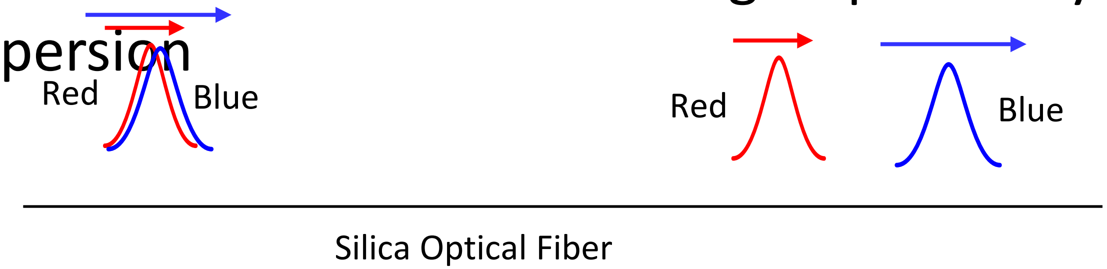
Group Velocity Dispersion



Mechanism for Optical Solitons

Group Velocity Dispersion in an Optical Fiber, near 1.55 μm

- Self-Phase modulation cancels group velocity dispersion



Self Phase Modulation