90 Degree Optical Hybrid

\[
\begin{bmatrix}
A(L) \\
B(L)
\end{bmatrix} = \begin{bmatrix}
\frac{1}{\sqrt{2}} & -j \frac{1}{\sqrt{2}} \\
-j \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix} \begin{bmatrix}
A(0) \\
B(0)
\end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & -j \\
-j & 1
\end{bmatrix} \begin{bmatrix}
A(0) \\
B(0)
\end{bmatrix}
\]

\[
A(L) = \frac{1}{\sqrt{2}}(A(0) - jB(0))
\]

\[
B(L) = \frac{1}{\sqrt{2}}(-jA(0) + B(0))
\]

\[
L = \frac{\pi}{(\beta_e - \beta_o)2}
\]

OPTI 500, Spring 2012, Lecture 20, Coherent Optical Receivers
Redefining the phase for input and output arms, we can write:

\[
\begin{bmatrix}
A_{out} \\
B_{out}
\end{bmatrix}
= \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & 1 \\
1 & -1
\end{bmatrix}
\begin{bmatrix}
A_{in} \\
B_{in}
\end{bmatrix}
\]

\[
A(L) = A_{out} = \frac{1}{\sqrt{2}}(A_{in} + B_{in})
\]

\[
B_{out} = jB(L) = \frac{1}{\sqrt{2}}(A_{in} - B_{out})
\]
Coherent Detection

\[ E_S(t) = A_s(t) \exp(j\omega_s t) \]

\[ E_{LO}(t) = A_{LO} \exp(j\omega_{LO} t) \]

\[ P_S = \left| A_s \right|^2 / 2 \] and \[ P_{LO} = \left| A_{LO} \right|^2 / 2 \]

From “Coherent Optical Communications: Historical Perspectives and Future Directions,” by Kazuro Kikuchi, Chapter 2 in High Spectral Density Optical Communications, Nakazawa et al. editors.
Coherent Detection

\[ E_1 = \frac{1}{\sqrt{2}} \left( E_S + E_{LO} \right) \]

\[ E_2 = \frac{1}{\sqrt{2}} \left( E_S - E_{LO} \right) \]
Coherent Detection

\[ I_1(t) = R \left[ \text{Re} \left\{ \frac{A_S(t) \exp(j \omega_S t) + A_{LO} \exp(j \omega_{LO} t)}{\sqrt{2}} \right\} \right]^{ms} \]

\[ = \frac{R}{2} \left[ P_S(t) + P_{LO} + 2 \sqrt{P_S(t) P_{LO}} \cos \left\{ \omega_{IF} t + \theta_{\text{sig}}(t) - \theta_{LO}(t) \right\} \right] \]

\[ I_2(t) = R \left[ \text{Re} \left\{ \frac{A_S(t) \exp(j \omega_S t) - A_{LO} \exp(j \omega_{LO} t)}{\sqrt{2}} \right\} \right]^{ms} \]

\[ = \frac{R}{2} \left[ P_S(t) + P_{LO} - 2 \sqrt{P_S(t) P_{LO}} \cos \left\{ \omega_{IF} t + \theta_{\text{sig}}(t) - \theta_{LO}(t) \right\} \right] \]

\[ R = \frac{\varepsilon \eta}{\hbar \omega_S} \]

\[ \theta_{\text{sig}} = \theta_s + \theta_n \]

\[ \omega_{IF} = \omega_s - \omega_{LO} \]
Down-Converted Signal

\[ I(t) = I_1(t) - I_2(t) = 2R\sqrt{P_s(t)P_{LO}} \cos \left( \omega_{IF} t + \theta_{\text{sig}}(t) - \theta_{LO}(t) \right) \]

Demodulating the IF Signal

(a) Envelope detection

(b) Differential detection

(c) Synchronous detection
Demodulating the IF Signal

We have three methods of demodulating $I_c(t)$, which are envelope (noncoherent) detection, differential (delay) detection, and synchronous (coherent) detection as shown in Fig. 2.13. In envelope detection, we measure $|I_c(t)|^2$ from Eq. (2.10), which gives us only the information on $P_s(t)$. Differential detection is effective for constant-envelope modulation formats such as $M$-ary PSK. In this scheme, we determine the phase difference between the current symbol and the previous one. In the synchronous detection scheme, although the total phase noise $\theta_n(t) = \theta_{sn}(t) - \theta_{LO}(t)$ might vary in time, the electrical phase-locked loop (PLL) can be used to estimate the phase noise and decode the symbol.