OPTI 500 B, Fall 2011, Take Home Exam

Open notes and open reference material.

Due November 9, 2011

1. The profile for the magnetic field for the m_{th} even TM mode in a slab waveguide is given by:

$$H_y^m(x) = \begin{cases} A_m \cos(\kappa_m d) e^{-\gamma_m (x-d)} & \text{in the upper clad } (x > d) \\ A_m \cos(\kappa_m x) & \text{in the core } (-d \le x \le d) \\ A_m \cos(-\kappa_m d) e^{\gamma_m (x+d)} & \text{in the lower clad } (x < -d) \end{cases}$$

   The power per unit length (in the y-direction) for a TM mode is given by:

$$P_m = \frac{\beta_m}{2\omega\varepsilon_0} \int_{-\infty}^{\infty} \frac{1}{n(x)^2} \left[H_y^m(x) \right]^2 dx,$$

   where n(x) is the refractive index.

   Find an expression for the amplitude coefficient A_m in terms of P_m, n_{core}, n_{clad}, κ_m, γ_m, β_m, and d.

2. Calculate the cut-off (minimum) wavelength for single mode operation for a fiber that has a core with a diameter of 7 micron and a refractive index of 1.458, and a cladding of refractive index 1.452.

3. Sketch the profile for an HE_{22} optical fiber mode indicating the magnitude and direction for both the electric and magnetic fields.

4. Use the result from problem 3 to sketch the profile for an LP_{12} optical fiber mode indicating the magnitude and direction for both the electric and magnetic fields.

5. Calculate the number of allowed modes in a multimode, step index fiber that has a core with a refractive index of 1.468 and diameter of 100 μm, and a cladding with a refractive index of 1.447, if the optical wavelength is 850 nm.