OPTI 500 B, Fall 2011, Take Home Exam

Solutions

1. The profile for the magnetic field for the m_{th} even TM mode in a slab waveguide is given by:

$$\mathrm{H}_y^m(x) = \begin{cases} A_m \cos(\kappa_m d) e^{-\gamma_m (x-d)} \text{ in the upper clad } (x > d) \\ A_m \cos(\kappa_m x) \text{ in the core } (-d \le x \le d) \\ A_m \cos(-\kappa_m d) e^{\gamma_m (x+d)} \text{ in the lower clad } (x < -d) \end{cases}$$

The power per unit length (in the y-direction) for a TM mode is given by:

$$P_m = \frac{\beta_m}{2\omega\varepsilon_0} \int_{-\infty}^{\infty} \frac{1}{n(x)^2} \left[H_y^m(x) \right]^2 dx,$$

where n(x) is the refractive index.

Find an expression for the amplitude coefficient A_m in terms of P_m, n_{core}, n_{clad}, κ_m, γ_m, β_m, and d.

$$P_m = \frac{\beta_m}{2\omega\varepsilon_0} \int_{-\infty}^{\infty} \frac{1}{n^2} \left[H_y^m(x) \right]^2 dx$$

$$\Rightarrow P_m = \frac{\beta_m}{2\omega\varepsilon_0} \int_{-\infty}^{-d} \frac{1}{n_2^2} \left[A_m \cos(-\kappa_m d) e^{\gamma_m (x+d)} \right]^2 dx$$

$$+ \frac{\beta_m}{2\omega\varepsilon_0} \int_{-d}^{d} \frac{1}{n_1^2} \left[A_m \cos(\kappa_m x) \right]^2 dx$$

$$+ \frac{\beta_m}{2\omega\varepsilon_0} \int_{d}^{\infty} \frac{1}{n_2^2} \left[A_m \cos(\kappa_m d) e^{-\gamma_m (x-d)} \right]^2 dx$$

$$\Rightarrow P_m = \frac{\beta_m}{2\omega\varepsilon_0} A_m^{\ 2} \left[\frac{\cos^2(\kappa_m d)}{n_2 2\gamma_m} + \frac{\sin(2\kappa_m d)}{n_1 2\kappa_m} + d + \frac{\cos^2(\kappa_m d)}{n_2 2\gamma_m} \right]$$

$$\Rightarrow \boxed{A_m = \left(\frac{P_m 2\omega\varepsilon_0}{\beta_m \left[\frac{\cos^2(\kappa_m d)}{n_2 \gamma_m} + \frac{\sin(2\kappa_m d)}{n_1 2\kappa_m} + d \right]} \right)^{1/2}}$$

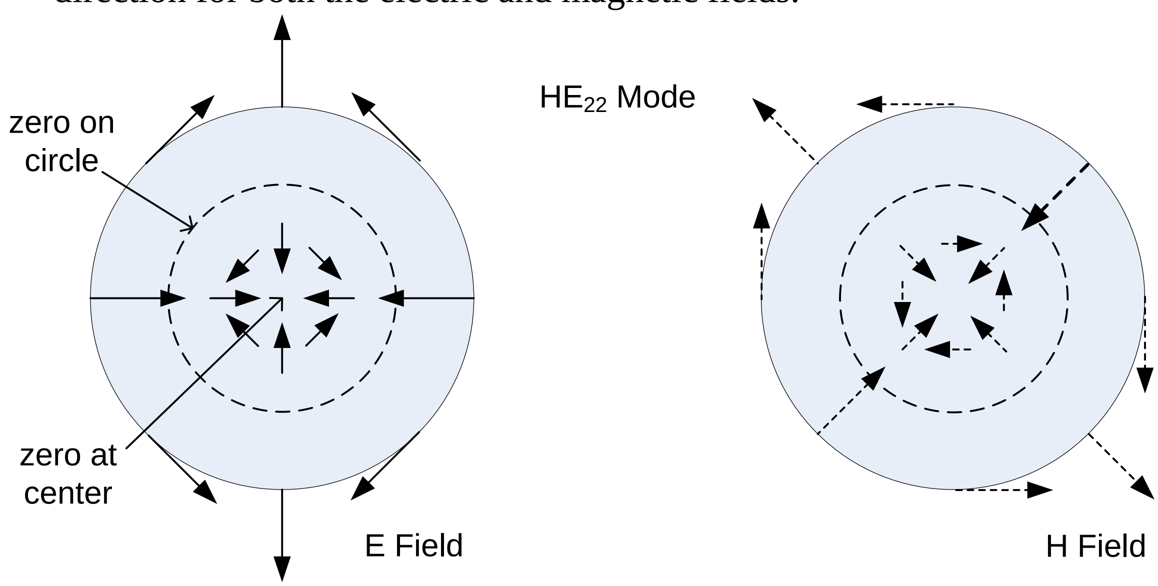
2. Calculate the cut-off (minimum) wavelength for single mode operation for a fiber that has a core with a diameter of 7 micron and a refractive index of 1.458, and a cladding of refractive index 1.452.

$$V = \frac{2\pi}{\lambda_0} a \sqrt{(n_{core})^2 - (n_{clad})^2}$$

cut-off when V=2.405 $\Rightarrow \lambda_0^{min} = \frac{2\pi}{2.404} a \sqrt{(n_{core})^2 - (n_{clad})^2}$

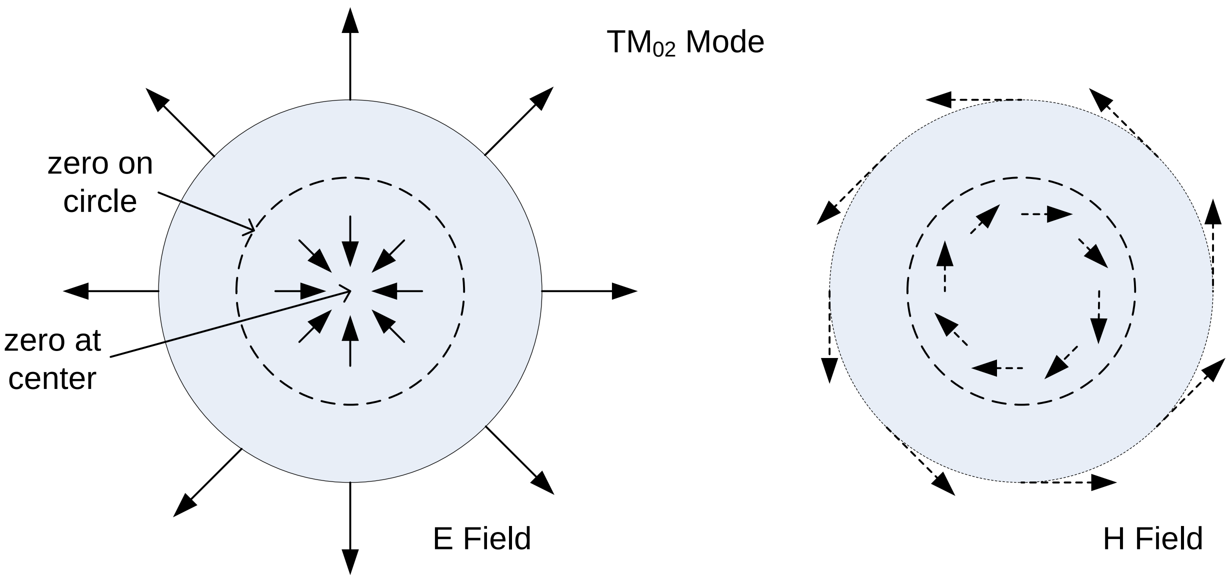
$$\Rightarrow \lambda_0^{min} = \frac{2\pi}{2.404}(3.5\mu m)\sqrt{(1.458)^2 - (1.452)^2} = 1.208\mu m$$

3. Sketch the profile for an HE_{22} optical fiber mode indicating the magnitude and direction for both the electric and magnetic fields.

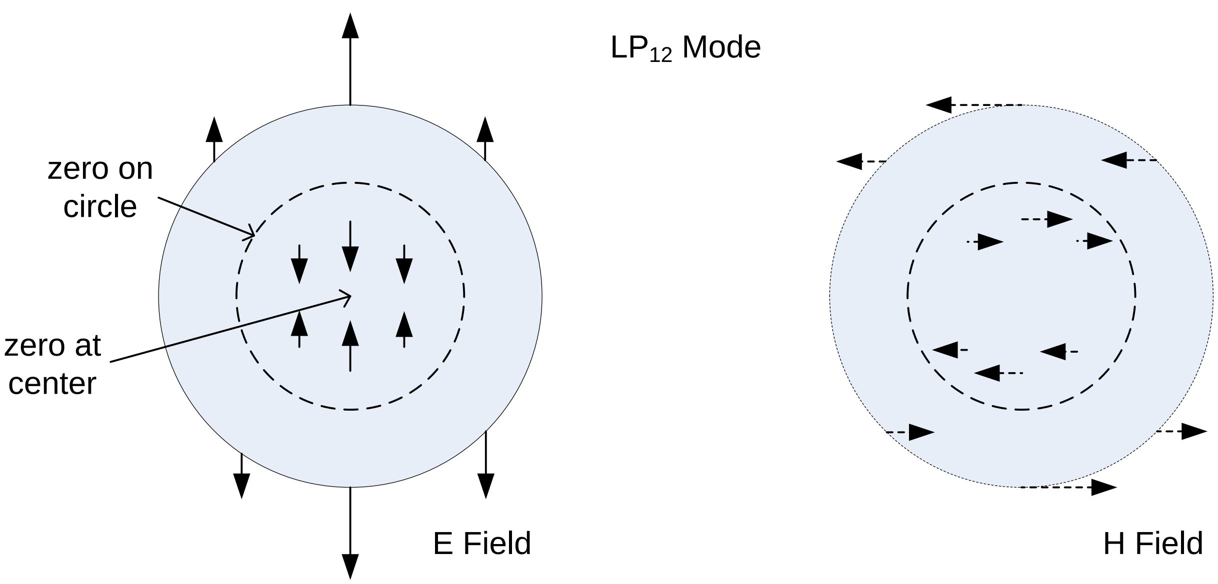


4. Use the result from problem 3 to sketch the profile for an LP_{12} optical fiber mode indicating the magnitude and direction for both the electric and magnetic fields.

Add



to the HE_{22} mode to get



5. Calculate the number of allowed modes in a multimode, step index fiber that has a core with a refractive index of 1.468 and diameter of 100 μm, and a cladding with a refractive index of 1.447, if the optical wavelength is 850 nm.

$$V = \frac{2\pi}{\lambda_0} a \sqrt{(n_{core})^2 - (n_{clad})^2} = \frac{2\pi}{0.85\mu m}(50\mu m)\sqrt{(1.468)^2 - (1.447)^2} = 91.445$$

$$\text{Number of Modes} \approx \frac{4V^2}{\pi^2} = \frac{4 \cdot 91.445^2}{\pi^2} = 3389$$