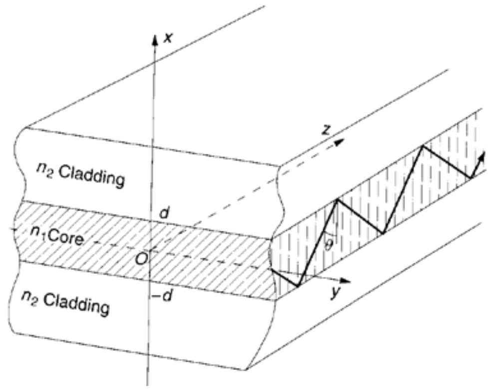
OPTI 500 A, Fall 2011
Homework #2
Fields for TE and TM Modes in Slab Waveguides
Solutions



$$\begin{aligned} \nabla \cdot \vec{D} &= \rho \\ \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} \end{aligned}$$ Maxwell's Equations

$$\begin{aligned} \vec{B} &= \mu \vec{H} \\ \vec{D} &= \varepsilon \vec{E} \end{aligned}$$ Constitutive Equations

Start with Maxwell's equations and the constitutive equations, assume fields are uniform in the y-direction, assume the electric permittivity and the magnetic permeability are constants, and derive the following relations for TE and TM modes for a slab waveguide.

Transverse Electric (TE) Mode

$E_z = 0$

$E_y = E_Y(x)$, $E_x = 0$

$$H_x = -\frac{\beta}{\omega\mu_0} E_y, \;\; H_z = \frac{j}{\omega\mu_0}\frac{dE_y}{dx}, \;\; H_y = 0$$

Transverse Magnetic (TM) Modes

$H_z = 0$,

$H_y = H_Y(x)$, $H_x = 0$

$$E_x = \frac{\beta}{\omega\varepsilon_0 n^2} H_y, \;\; E_z = -\frac{j}{\omega\varepsilon_0 n^2}\frac{dH_y}{dx}, \;\; E_y = 0$$

Hint: The x, y, and z components of Faraday's and Ampere's Equations can be written.

$$\frac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu_0 H_x \quad \boxed{F1}$$

$$-j\beta E_x + \frac{\partial E_z}{\partial x} = -j\omega\mu_0 H_y \quad \boxed{F2}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu_0 H_z \quad \boxed{F3}$$

$$\frac{\partial H_z}{\partial y} + j\beta H_y = j\omega\varepsilon_0 n^2 E_x \quad \boxed{A1}$$

$$-j\beta H_x + \frac{\partial H_z}{\partial x} = j\omega\varepsilon_0 n^2 E_y \quad \boxed{A2}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon_0 n^2 E_z \quad \boxed{A3}$$

$E_z = 0,$ by definition, for TE Modes. We also assmume $\frac{\partial}{\partial y}$ gives 0.

Then from $\boxed{A3}$ we have $\frac{\partial H_y}{\partial x} = 0.$

This means $H_y = const,$ and for confined modes we mus choose $\boxed{H_y = 0}$.

Using $\boxed{F2}$ together with $E_z = 0$ and $H_y = 0$ gives $\boxed{E_x = 0}$.

$\boxed{A1}$ can be written $0 + j\beta H_y = j\omega\varepsilon_0 n^2 E_x,$which gives $\boxed{E_x = \frac{\beta}{\omega\varepsilon_0 n^2} H_y}$.

$\boxed{F3}$ can be written $\frac{\partial E_y}{\partial x} - 0 = -j\omega\mu_0 H_z,$ which gives $\boxed{H_z = \frac{j}{\omega\mu_0}\frac{\partial E_y}{\partial x}}$.

$H_z = 0,$ by definition, for TM Modes. We also assmume $\frac{\partial}{\partial y}$ gives 0.

The derivations for TM modes are the same as above with
E replaced by H, and H replaced by E.