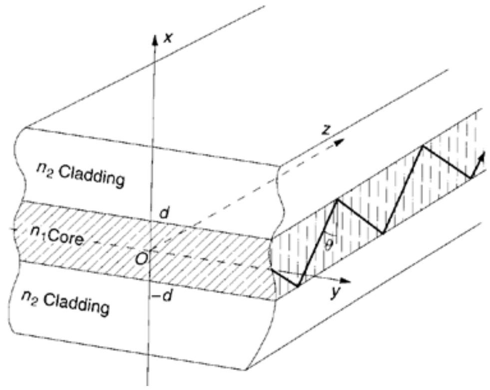
OPTI 500 A, Fall 2011
Homework #2
Fields for TE and TM Modes in Slab Waveguides
Due October 5th, 2011



$$\nabla \cdot \vec{D} = \rho$$
$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Maxwell's Equations

$$\vec{B} = \mu \vec{H}$$
$$\vec{D} = \varepsilon \vec{E}$$

Constitutive Equations

Start with Maxwell's equations and the constitutive equations, assume fields are uniform in the y-direction, assume the electric permittivity and the magnetic permeability are constants, and derive the following relations for TE and TM modes for a slab waveguide.

Transverse Electric (TE) Mode

$E_z = 0$
$E_y = E_Y(x)$, $E_x = 0$

$$H_x = -\frac{\beta}{\omega\mu_0}E_y,\ \ H_z = \frac{j}{\omega\mu_0}\frac{dE_y}{dx},\ \ H_y = 0$$

Transverse Magnetic (TM) Modes

$H_z = 0$,
$H_y = H_Y(x)$, $H_x = 0$

$$E_x = \frac{\beta}{\omega\varepsilon_0 n^2}H_y,\ \ E_z = -\frac{j}{\omega\varepsilon_0 n^2}\frac{dH_y}{dx},\ \ E_y = 0$$

Hint:

The x, y, and z components of Faraday’s and Ampere’s Equations can be written.

$$\begin{cases} \dfrac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu_0 H_x \\ -j\beta E_x - \dfrac{\partial E_z}{\partial x} = -j\omega\mu_0 H_y \\ \dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} = -j\omega\mu_0 H_z \end{cases}$$

$$\begin{cases} \dfrac{\partial H_z}{\partial y} + j\beta H_y = j\omega\varepsilon_0 n^2 E_x \\ -j\beta H_x - \dfrac{\partial H_z}{\partial x} = j\omega\varepsilon_0 n^2 E_y \\ \dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y} = j\omega\varepsilon_0 n^2 E_z. \end{cases}$$